Optimal Minimum Wage Policy in Competitive Labor Markets*

David Lee, Princeton University and NBER

Emmanuel Saez, UC Berkeley and NBER

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Abstract

This paper provides a theoretical analysis of optimal minimum wage policy in a perfectly competitive labor market. We show that a binding minimum wage – while leading to unemployment – is nevertheless desirable if the government values redistribution toward low wage workers. This result remains true in the presence of optimal nonlinear taxes and transfers. In that context, a minimum wage effectively rations low skilled labor which is subsidized by the optimal tax/transfer system, and improves upon the second-best tax/transfer optimum. When labor supply responses are along the extensive margin, a minimum wage and work subsidies are complementary policies, and the co-existence of minimum wages with positive tax rates for low skilled workers is (second-best) Pareto inefficient. We derive formulas for the optimal minimum wage (with or without optimal taxes) as a function of the elasticities of labor supply and demand and the redistributive tastes of the government. The optimal minimum wage decreases with the demand elasticity for low skilled labor but increases with the supply elasticity of low skilled labor. The optimal minimum wage follows an inverted U-shape as a function of the degree of redistributive tastes of the government.

*David Lee, Department of Economics, Princeton University, Emmanuel Saez, University of California, Department of Economics, 549 Evans Hall #3880, Berkeley, CA 94720, saez@econ.berkeley.edu. We thank John Bound, Pierre Cahuc, David Card, Guy Laroque, Etienne Lehmann, and numerous seminar participants for useful discussions and comments.

1 Introduction

The minimum wage is a widely used but controversial policy tool. Most OECD countries have adopted minimum wage policies over the course of the 20th century. In a number of OECD countries, and especially in continental Europe, the minimum wage binds for a non-trivial share of the workforce (see e.g., OECD 1998, Immervoll, 2007). Minimum wages are a potentially useful tool for redistribution because they increase low skilled workers' wages at the expense of other factors of production such as higher skilled labor or capital. They may, however, also lead to involuntary unemployment and hence worsen the welfare of workers who lose their jobs because of the minimum wage. An enormous empirical literature has studied the extent to which minimum wages affect the wages and employment of low-skilled workers (see e.g., Brown et al. 1982, Card and Krueger 1995, Dolado et al. 1996, Brown 1999, or Neumark and Wascher 2006 for extensive surveys). The normative literature on the minimum wage, however, is much less extensive.

This paper provides a normative analysis of the optimal minimum wage in a conventional competitive labor market model, using the standard social welfare framework adopted in the optimal tax theory literature following the seminal contributions of Diamond and Mirrlees (1971) and Mirrlees (1971). In most of our analysis, we adopt the important "efficient rationing" assumption – that unemployment induced by the minimum wage hits workers with the lowest surplus first.[1] Our goal is to use this framework to make explicit the trade-offs involved when a government sets a minimum wage, and to shed light on the appropriateness of a minimum wage in the context of optimal taxes and transfers.

The first part of the paper considers a competitive labor market with no taxes and transfers. Although unrealistic, this case has the advantage of simplicity and it transparently shows the key forces at play when choosing a minimum wage rate. We are thus formalizing the pros and cons of the minimum wage that have been discussed primarily in an informal way in the labor literature (see e.g. Stigler 1946 for an early analysis and Freeman 1996 and Dolado, Felgueroso, and Jimeno 2000, for recent expositions). We show that a binding minimum wage is desirable as long as 1) the government places a non-zero value on redistribution from high-

[1] Although, we believe that efficient rationing is the most natural assumption, we also discuss in detail how our results are modified if unemployment hits low skilled workers independently of surplus, what we call "uniform rationing".

to low wage workers, 2) the demand elasticity of low-skilled labor is finite, and 3) the supply elasticity of low-skilled labor is positive.

We then derive a formula for the optimal minimum wage as a function of those demand and supply elasticities, and the redistributive tastes of the government. Unsurprisingly, the optimal minimum wage is decreasing in the demand elasticity because a minimum wage has larger unemployment effects when the demand elasticity is higher. The optimal minimum wage is increasing in the supply elasticity because a high supply elasticity implies that workers have a low surplus from working (since many would leave the labor force if the wages were slightly reduced). The size of the optimal minimum wage follows an inverted U-shape with the degree of the government's redistributive tastes: there is no role for the minimum wage if the government does not value redistribution, and if the government has extreme Rawlsian preferences, then the costs of involuntary unemployment dominate the value of transfers to low skilled workers.

The second part of the paper considers how the results change when the government also uses taxes and transfers to achieve redistributive goals. As described below in more detail, our key innovation is to abstract from the hours of work decision and focus only in the job choice and work participation decision. In that context, the government observes only occupation choices and the corresponding wage but not the utility work costs incurred by individuals. Therefore, the informational constraints that the government faces when imposing a minimum wage policy and a nonlinear tax/transfer system are well defined and mutually consistent. In such a model, we show that a minimum wage is desirable as long as the government values redistribution toward low skilled workers (and assuming again that the low skilled work demand elasticity is finite and the supply elasticity is positive). Such a situation is very likely to hold in practice suggesting that the minimum wage is a useful tool to supplement taxes and transfers for redistribution purposes. This result can be seen as an application of the Guesnerie (1981) and Guesnerie and Roberts (1984) theory of quantity controls in second best economies: When the government values redistribution toward low skilled workers, the optimal tax/transfer system over-encourages the supply of low skilled labor. In that context, a minimum wage effectively rations over-supplied low skilled labor which is socially desirable. Put in another way, with a minimum wage rationing low skilled jobs, the government can increase redistribution toward those low skills workers without inducing any adverse supply response.

When labor supply responses are along the participation margin, we show that a minimum wage should always be associated with work subsidies (negative tax rates), and that imposing positive tax rates on work on minimum wage workers is second-best Pareto inefficient. Cutting taxes on low income workers while reducing the (pre-tax) minimum wage leads to a Pareto improvement. This latter result remains true even if rationing is not efficient.

We derive optimal formulas for the jointly optimal tax/transfer system and minimum wage. The formulas as well as numerical simulations show that – as in the basic case without taxes and transfers – the optimal minimum wage with optimal taxes continues to be decreasing in the demand elasticity for low skilled work, increasing in the supply elasticity for low skilled work, and follows an inverted U-shape pattern with respect to the strength of redistributive tastes. Numerical simulations also show that, with an optimum minimum wage, the optimum subsidy for low skilled workers is larger than the optimum subsidy in the absence of the minimum wage.

The remainder of the paper is organized as follows. Section 2 provides an overview of the existing literature most relevant to our analysis. Section 3 presents the basic two skill model with extensive labor supply responses and analyzes optimal minimum wage policy in a situation with no taxes. Section 4 introduces taxes and transfers and analyzes joint optimal minimum wage policy and taxes and transfers. Section 5 presents various extensions such as "uniform rationing", general labor supply responses, and many skills and goods. Section 6 offers a brief conclusion.

2 Existing Literature

We begin by describing how our analysis relates to the existing normative discussions and analyzes of the minimum wage. The basic point that a large demand elasticity for low skilled workers implies that the negative employment effects of a minimum wage will be large has been recognized for a long time. A well-known related point is that, if the demand elasticity is larger than one in absolute value, then a minimum wage reduces total pay going to low skilled workers (see e.g., Freeman 1996 or Danziger, 2006 for recent expositions). By contrast, our analysis reveals no special significance to the absolute demand elasticity being one, and additionally highlights the importance of labor supply elasticities.

We can divide the existing normative literature on the minimum wage into two strands. One literature, most closely associated with labor economics, focuses on efficiency effects of the minimum wage in the presence of labor market imperfections. It is well known, at least since Robinson (1933), that if the labor market is monopsonistic, then a minimum wage can actually increase both employment and low skilled wages and hence improve efficiency (see e.g., Card and Krueger 1995 or Manning 2003 for recent expositions). A number of papers have shown that the monopsony logic for the desirability of the minimum wage extends to other models of the labor market with frictions or informational asymmetries such as efficiency wages (Drazen, 1986, Jones, 1987, Rebitzer and Taylor, 1995), bargaining models (Cahuc, Zylberberg, and Saint-Martin, 2001), signalling models (Lang, 1987), search models (Swinnerton, 1996, Acemoglu 2001, Flinn, 2006), or endogenous growth models (Cahuc and Michel, 1996). These studies focus on efficiency and generally abstract from the government's redistributive goals, and do not consider the role of the minimum wage when taxes and transfers are available to achieve these goals.

A second smaller literature in public economics has investigated whether the minimum wage is desirable for redistributive reasons and especially in situations where the government can also use taxes and transfers for redistribution. Most models have adopted the standard Mirrlees (1971) model on optimal taxation or the discrete version developed by Stiglitz (1982). The general principle, following Allen (1987) and Guesnerie and Roberts (1987), is that a minimum wage is desirable if it expands the redistributive power of the government by relaxing incentive compatibility constraints. In the context of the two-skill Stiglitz (1982) model, Allen (1987) and Guesnerie and Roberts (1987) show that a minimum wage can sometimes usefully supplement an optimal linear tax but is never useful to supplement an optimal nonlinear tax. This result is obtained because a minimum wage does not prevent in any way high skilled workers from imitating low skilled workers in the Stiglitz (1982) model. This is in contrast to our occupational model, and we later return to this important difference.

By contrast, Marceau and Boadway (1994) and Boadway and Cuff (2001) develop models where the minimum wage helps relax the incentive constraints that the government face, and show that, in that context, a minimum wage is actually desirable. Marceau and Boadway (1994) add the unemployment insurance tool to the Allen (1987) model. As only low skilled workers can become unemployed because of the minimum wage, the unemployment insurance can efficiently screen low skilled workers. Boadway and Cuff (2001), using a continuum of skills model as in Mirrlees (1971), show that a minimum wage policy implicitly allows the government to observe skills at the bottom of the distribution and this feature can be exploited to improve upon the standard Mirrlees (1971) allocation.

As recognized by Guesnerie and Roberts (1987), these contrasting results stem from informational inconsistencies that arise when a minimum wage is introduced: The minimum wage implementation requires observing the wage rates while the income tax is based on earnings because wage rates and hours of work are not observable separably for tax purposes. If wage rates are directly observable, then the government can achieve any first best allocation by conditioning taxes and transfers on immutable wage rates (and obviously, no minimum wage would be needed).

The negative results on the desirability of the minimum wage of Allen (1987) appear in an environment where the government implicitly observes the wage rates for low-skilled workers – a necessity when implementing a minimum wage – yet ignores this extra information in choosing the income tax. On the other hand, the positive results of Marceau and Boadway (1994) and Boadway and Cuff (2001) are not driven by the use of the minimum wage *per se*, but rather because the government uses the *information on wage rates* that is created by the minimum wage.[2]

As mentioned in introduction, for our analysis of the optimal combination of taxes and transfers and a minimum wage, we resolve this informational inconsistency by abstracting from the hours of work decision and focusing only on job choice and work participation decisions. We consider a government facing a single set of informational constraints – occupation choices and corresponding wages are observed while the disutility of work faced by each individual is not. This allows us to consider whether or not, facing a fixed informational constraint, the additional policy lever of a minimum wage can improve upon the second best allocation when only taxes and transfers are used.

Finally, a number of studies have brought together those two literature strands and explored the issue of joint optimal minimum wages and optimal taxes and transfers in imperfect

[2]Some papers have actually explicitly modelled limitations on the use of taxes and transfers using political economy arguments. In that context, a minimum wage can be a useful tool for redistribution (see e.g., Dreze and Gollier 1993 and Bacache and Lehmann 2005).

labor markets. Blumkin and Sadka (2005) consider a signalling model where employers do not observe productivities perfectly and show that a minimum can be desirable to supplement the optimal tax system in that context. Cahuc and Laroque (2007) show that, in a monopsonistic labor market model, with participation labor supply responses only, the minimum wage should not be used when the government can use optimal nonlinear income taxation.[3] Hungerbuhler and Lehmann (2007) analyze a search model and show that a minimum wage can improve welfare even with optimal income taxes if the bargaining power of workers is sufficiently low. There, however, if the government can directly increase the bargaining power of workers, then the desirability of the minimum wage vanishes. These latter two papers are closest to ours because they also abstract from the hours of work choice and consider only the participation margin for labor supply. Our analysis, however, considers the simpler case of perfect competition with no market frictions. Therefore, we see our contribution as complementary to those of Cahuc and Laroque (2007) and Hungerbuhler and Lehmann (2007).

3 Optimal Minimum Wage with no Taxes/Transfers

3.1 Basic Model

• Demand Side

We consider a simple two sector model where production $F(h_1, h_2)$ depends on the number of low skilled workers h_1 and the number of high skilled workers h_2.[4] We assume perfectly competitive markets so that firms take the wages (w_1, w_2) as given. The production sector chooses labor demand (h_1, h_2) to maximize profits: $\Pi = F(h_1, h_2) - w_1 h_1 - w_2 h_2$, which leads to the standard first order conditions where wages are equal to marginal product:

$$w_i = \frac{\partial F}{\partial h_i}, \tag{1}$$

for $i = 1, 2$. We assume that, in any equilibrium, $w_1 < w_2$. We also assume constant returns to scale so that there are no profits in equilibrium: $\Pi = F - w_1 h_1 - w_2 h_2 = 0$. We denote by

[3]Importantly, Cahuc and Laroque (2007) also briefly mention the competitive case, recalling the results of Allen (1987), and conjecturing that minimum wages are only useful in competitive environments when tax schemes are constrained. Below, however, we show – contrary to Allen (1987) – within our simple two-skill model that even with an unconstrained nonlinear optimal tax scheme, the minimum wage can still lead to improvements.

[4]We show below that the model can be generalized to include a capital input, more than two labor inputs, and more than one consumption output good.

$\sigma = d\log(h_1/h_2)/d\log(w_2/w_1)$ the elasticity of substitution between high and low skills in the production function.[5]

- **Supply Side**

We assume that each individual is either low skilled or high skilled. We normalize the population of workers to one and denote by h_1^0 and h_2^0 the fraction of low and high skilled with $h_1^0 + h_2^0 = 1$. We assume that each worker faces a cost of working θ representing disutility of work. In order to generate smooth supply curves, we assume that θ is distributed according to smooth cumulated distributions $P_1(\theta)$ and $P_2(\theta)$ for low and high skill workers respectively. There are three groups of individuals: group 0 for workers (either low or high skilled) out of work (earning 0), group 1 for low skilled workers (earning w_1), and group 2 for high skilled workers (earning w_2). We denote by h_i the fraction of individuals in each group $i = 0, 1, 2$.

In this section, we assume that there are no taxes and transfers. For simplicity and throughout the paper, we assume no income effects in the labor supply decision.[6] An individual with skill i and cost of work θ makes its binary labor supply decision $l = 0, 1$ in order to maximize utility $u = w_i \cdot l - \theta \cdot l$. Therefore, $l = 1$ if and only if $\theta \leq w_i$. Hence, the aggregate labor supply functions for $i = 1, 2$ are:

$$h_i = h_i^0 \cdot P_i(w_i). \tag{2}$$

We denote by e_i the elasticity of labor supply h_i with respect to the wage rate w_i:

$$e_i = \frac{w_i}{h_i}\frac{\partial h_i}{\partial w_i} = \frac{w_i \cdot p_i(w_i)}{P_i(w_i)},$$

where p_i is the density distribution of θ.

Combining the demand and supply side equations (1) and (2) defines a single undistorted competitive equilibrium denoted by $(w_1^*, w_2^*, h_1^*, h_2^*)$.

3.2 Desirability of the Minimum Wage

We depict on Figure 1 the competitive equilibrium for low skilled labor using standard supply and demand curve representation. The supply curve is defined as $h_1 = h_1^0 P_1(w_1)$. Because of constant returns to scale in production, only the ratio h_1/h_2 is well defined on the demand side.

[5] For example, this elasticity is constant in the case of CES production function $F(h_1, h_2) = (a_1 h_1^{(\sigma-1)/\sigma} + a_2 h_2^{(\sigma-1)/\sigma})^{\sigma/(\sigma-1)}$, which we will use in numerical simulations.

[6] The presence of income effects would not change our key results.

For our purposes, we define the partial equilibrium demand for low skilled work $h_1 = D_1(w_1)$ as follows. $D_1(w_1)$ is the level of demand when w_1 is set exogenously by the government (such as with a minimum wage policy) and (h_2, w_2) is defined as the market clearing equilibrium on the high skilled labor market. Therefore, Figure 1 captures general equilibrium effects as well.[7]

Starting from the market equilibrium $(w_1^*, w_2^*, h_1^*, h_2^*)$ and as illustrated on Figure 1, we introduce a *small* minimum wage just above the low skill wage w_1^*, which we denote by $\bar{w} = w_1^* + d\bar{w}$. Formally, the small minimum wage creates changes dw_1, dw_2, dh_1, dh_2 in our key variables of interest. By definition, $dw_1 = d\bar{w}$.

From $\Pi = F(h_1, h_2) - w_1 h_1 - w_2 h_2$, we have $d\Pi = \sum_i [(\partial F/\partial h_i) dh_i - w_i dh_i - h_i dw_i] = -h_1 dw_1 - h_2 dw_2$ using (1). The no profit condition $\Pi = 0$ implies that $d\Pi = 0$ and hence:

$$h_1 dw_1 + h_2 dw_2 = 0. \quad (3)$$

Equation (3) is fundamental and shows that the earnings gain of low skilled workers $h_1 dw_1 > 0$ due to the small minimum wage is exactly compensated by an earnings loss of high skilled workers $h_2 dw_2 < 0$. If the government values redistribution from high skilled workers to low skilled workers, such a transfer is socially desirable.[8]

However, in addition to this transfer, the minimum wage also creates involuntary unemployment. As depicted on Figure 1, the minimum wage creates excess supply so that equation $h_1 = h_1^0 P_1(w_1)$ no longer holds and h_1 is defined by the demand equation $h_1 = D_1(\bar{w})$. Hence, employment is reduced by $dh_1 = -\eta_1 h_1 d\bar{w}/w_1$ where $\eta_1 = -(w_1/h_1)D'_(w_1)$ is the low skilled labor demand elasticity (defined with a minus sign so that $\eta_1 > 0$).

Formally, we can derive an expression for η_1 as follows. Using the definition of the substitution elasticity σ, we have $dh_2/h_2 - dh_1/h_1 = \sigma(dw_1/w_1 - dw_2/w_2)$ and using the supply equation for high skilled labor: $dh_2/h_2 = e_2 dw_2/w_2$, and from (3), the fact that $h_2 dw_2 = -h_1 dw_1$, we have: $\eta_1 = \sigma + (e_2 + \sigma)(h_1 w_1)/(h_2 w_2)$. When the low skilled sector is small (relative to the other sectors), the second term is small, and the demand elasticity is closely approximated by

[7] For example, in the case of a CES production function $F(h_1, h_2) = (a_1 h_1^{(\sigma-1)/\sigma} + a_2 h_2^{(\sigma-1)/\sigma})^{\sigma/(\sigma-1)}$, the ratio of the demand side equations (1) implies that $h_1 = h_2 \cdot (a_1/a_2)^\sigma \cdot (w_2/w_1)^\sigma$. The no profit condition $F = w_1 h_1 + w_2 h_2$ implies that $a_1^\sigma w_1^{1-\sigma} + a_2^\sigma w_2^{1-\sigma} = 1$, which defines $w_2(w_1)$ as a function of w_1. The supply equation $h_2 = h_2^0 P_2(w_2)$ then defines $h_2(w_1)$ as a function w_1. Therefore, we have $D_1(w_1) = h_2(w_1) \cdot (a_1/a_2)^\sigma \cdot (w_2(w_1)/w_1)^\sigma$.

[8] More generally, with more labor inputs and capital inputs, the loss will be distributed among all the other production inputs, with the closest substitutes to low skilled workers carrying most of the burden (see below).

the substitution elasticity σ. To evaluate the welfare cost of the involuntary unemployment, we will make the important assumption of *efficient rationing*.

Assumption 1 ***Efficient Rationing****: Workers who involuntary lose their job because of the minimum wage are those with the highest costs of work (and hence those with the least surplus from working).*

Although the standard "textbook" supply and demand model is silent on *which* employees become involuntarily unemployed due to the minimum wage, there are three conceptual reasons why it is arguably plausible that job loss occurs among those with the least surplus. First, presumably workers with the most surplus would exert the most effort to keep their jobs. Indeed, a standard Coasian argument implies that a worker with low surplus would be willing to let an unemployed worker with high surplus from working take her job in exchange for a private transfer, leading to efficient rationing overall. Second, to the extent that turnover is costly, the employer may seek to first lay off workers who are least likely to be a stable employee (i.e., those with low surplus from the job). Third, in markets where there is high turnover, and employers adjust employment levels over time by ceasing hiring, and allowing natural attrition, presumably the lowest surplus workers would quit first.[9] This is probably a good approximation of the low skilled US labor market (see e.g., Card and Krueger 1995) although this might not hold in European countries where turnover is lower.

In the end, which workers loose their job because of the minimum wage is an empirical question. Unfortunately, empirical work on this question is thin. Two types of evidence can be brought to bear on the issue. First, empirical studies on the effects of the minimum wage in the United States has shown that increases in the minimum wages reduces turnover (see e.g., Dube, Naidu, and Reich 2007 for a recent study in the case of citywide minimum wages). The reduction in turnover implies that minimum wage workers derive on average higher surplus when the minimum wage increases. Obviously, the increase in average surplus of the remaining workers due to a minimum wage increase is larger when those laid out have the least surplus. Therefore, this empirical evidence is, at least in the United States, consistent with efficient rationing. Second, Luttmer (2007) has proposed a recent direct empirical test of efficient

[9]Those last two arguments are based on search friction arguments which could possibly be incorporated in the model explicitly using search modelling. Hungerbuhler and Lehmann (2007) propose an analysis in that direction.

rationing in the United States using variation in minimum wage across the US states and finds support for the assumption that the minimum wage does not lead to inefficient allocation of labor.[10] Obviously, the case with efficient rationing is the most favorable to minimum wage policy, a point to which we will return when we present alternative results assuming that unemployment losses are distributed independently of surplus.

Under efficient rationing, as can be seen on Figure 1, the welfare loss due to involuntary unemployment due to the minimum wage is second order (exactly as in the standard Harberger deadweight burden analysis). Therefore, we can state our first result.

Proposition 1 *With no taxes and transfers and under the efficient rationing assumption 1, introducing a minimum wage is desirable if (1) the demand elasticity for low skilled workers is finite, (2) the supply elasticity of low skilled workers is positive, and (3) the government values redistribution from high skilled workers toward low skilled workers.*

It is useful to analyze briefly the desirability of the minimum wage when either of those three conditions breaks down. Obviously, condition (3) is necessary because if the government does not value the transfer created by the minimum wage, the minimum wage only creates deadweight burden and is therefore not desirable. If the government values redistribution, this condition naturally holds, except in the extreme case where the government has Rawlsian preferences and cares only about those out of work and hence values equally (at zero) marginal income to low and high skilled workers.

Condition (1) is also necessary. If the demand elasticity is infinite (which in the context of our simple model is equivalent to assuming that low and high skill workers are perfect substitutes, $\sigma = \infty$ and hence $F = a_1 h_1 + a_2 h_2$), then any minimum wage set above $w_1^* = a_1$ will shut down entirely the low skilled labor market and hence cannot be desirable. A large body of empirical work has shown that the demand elasticity for low skilled labor is not infinite (see e.g. Hamermesh (1996) for a survey). Related, evidence of a spike in the wage density distribution at the minimum wage also implies a finite demand elasticity (Card and Krueger 1995).

When condition (2) breaks down and the supply elasticity is zero, then there are no marginal workers with no surplus from working. Therefore, the unemployment welfare loss

[10] This is in contrast to a situation with low turnover as in the housing market with rent control as in Glaeser and Luttmer (2003).

is no longer second order. In that context, whether a minimum wage is desirable depends on the parameters of the model (reservation wages of low skilled workers and the size of the demand elasticity).[11] Empirically, a large body of work has shown that there are substantial participation supply elasticities for low skilled workers (see e.g., Blundell and MaCurdy, 1999 for a survey).

Finally, when the efficient rationing assumption does not hold, then the case for the minimum wage depends on the degree of inefficiency in the rationing along with the other parameters of the model.[12]

3.3 Derivation in the Basic Model

Let us now derive the optimal minimum wage when the conditions of Proposition 1 are met. As displayed in Figure 2, with a non infinitesimal minimum wage $\bar{w} > w_1^*$, we can define $\underline{w}$ as the reservation wage (or equivalently the cost of work) of the marginal low skilled worker (i.e., the worker getting the smallest surplus from working). Formally, $\underline{w}$ is defined so that $h_1^0 P_1(\underline{w}) = D_1(\bar{w})$ where $D_1(\bar{w})$ is the demand function for low skilled work when the minimum wage is $\bar{w}$.

We assume that the government evaluates outcomes using a utilitarian social welfare function of the form: $SW = \int G(u)d\nu$ where $u \to G(u)$ is an increasing and concave function. The concavity of $G(.)$ can represent either the individuals risk aversion and/or the redistributive tastes of the government. Given the structure of our basic model, we can write social welfare as:

$$SW = (1 - h_1 - h_2)G(0) + h_1^0 \cdot \int_0^{\underline{w}} G(\bar{w} - \theta)p_1(\theta)d\theta + h_2^0 \cdot \int_0^{w_2} G(w_2 - \theta)p_2(\theta)d\theta. \quad (4)$$

The government picks $\bar{w}$ to maximize SW subject to the constraints that $w_i = \partial F/\partial h_i$ for $i = 1, 2$, the no profit condition $h_1 w_1 + h_2 w_2 = F(h_1, h_2)$ and $h_2 = h_2^0 P_2(w_2)$. In order to obtain a first order condition for the optimal minimum wage $\bar{w}$, we consider a small change $d\bar{w}$ around $\bar{w}$. Figure 2 shows that this change has two effects.

First, it creates a transfer $h_1 d\bar{w}$ toward low skilled workers at the expense of high skilled workers (as $h_2 dw_2 = -h_1 d\bar{w}$). The net social value of this transfer is $dT = [g_1 - g_2]h_1 d\bar{w}$ where

[11] The well known result that a minimum wage cannot be desirable if $\eta_1 > 1$ is based on such a model with fixed labor supply.

[12] We explore a case with inefficient rationing in Section 3.4.

$g_i = \int G'(w_i - \theta) p_i d\theta / P_i$ the average social marginal welfare weight of workers of skill i.

Second, the minimum wage increases involuntary unemployment by $dh_1 = -\eta_1 h_1 d\bar{w}/\bar{w}$. Using the efficient rationing assumption, those marginal workers have a reservation wage equal to $\underline{w}$. Therefore, each worker becoming unemployed generates a social welfare cost equal to $G(\bar{w}-\underline{w})-G(0)$. We can define $g_0^e = [G(\bar{w}-\underline{w})-G(0)]/(\bar{w}-\underline{w})$ as the marginal welfare weight put on earnings lost due to unemployment. Thus, the welfare cost due to unemployment is $dU = -g_0^e \cdot (\bar{w} - \underline{w}) \cdot \eta_1 \cdot h_1 d\bar{w}/\bar{w}$.

Note that the change $dh_2 < 0$ does not generate welfare effects because marginal workers in the high skill sector have no surplus from working and hence the welfare cost is second order. At the optimum, we have $dT + dU = 0$, which implies:

$$\frac{\bar{w} - \underline{w}}{\bar{w}} = \frac{g_1 - g_2}{\eta_1 \cdot g_0^e}. \tag{5}$$

Formula (5) shows that the optimum minimum wage wedge (defined as $(\bar{w} - \underline{w})/\bar{w}$) is decreasing in the labor demand elasticity η_1 as a higher elasticity creates larger negative unemployment effects. As we saw above, when $\eta_1 = \infty$, formula (5) shows that there should be no minimum wage.

The optimum wedge is increasing with $g_1 - g_2$ which measures the net value of transferring \$1 from high skilled workers to low skilled workers,[13] and decreasing in g_0^e which measures the social cost of earnings losses due to involuntary unemployment. Obviously g_0^e, g_1, and g_2 are endogenous parameters and depend on the primitive social welfare function $G(.)$ but also on the level of the minimum wage. At the optimum, however, we have $g_a^0 \geq g_1 \geq g_2$. Increasing the redistributive tastes of the government by choosing a more concave $G(.)$ function has an ambiguous effect on the level of the optimum $\bar{w}$ because it is likely to increase both $g_1 - g_2$ and g_a^0. If the government does not value redistribution at all, then $G(u) = u$ and hence $g_1 = g_2 = g_a^0$ and there should be no minimum wage ($\bar{w} = \underline{w}$ from formula (5)). If the government has extreme redistributive tastes with a Rawlsian objective, then $g_1 = g_2 = 0$ and $g_0^e > 0$ which also implies that there should be no minimum wage in that case as well. Therefore, the level of the optimum $\bar{w}$ is expected to follow an inverted U-shape with the level of redistributive tastes.

[13] It is theoretically possible to find distributions P_1 and P_2 such that $g_1 < g_2$ at the market optimum: for example if high skilled workers have much higher costs of working than low skilled workers. In that case, there should be no minimum wage. This situation is ruled out implicitly by condition (3) in proposition 1 and it would be easy to provide primitive conditions on P_1 and P_2 to ensure that this condition is met.

Formula (5) is not an explicit formula because it depends on $\underline{w}$ which itself depends on $\bar{w}$ through the supply function (as illustrated on Figure 2). It is possible, however, to obtain a more explicit formula if we are willing to make assumptions about the shape of the demand and supply functions. The simplest assumption is that the elasticities of demand η_1 and supply e_1 are constant. In that case $D(w_1) = D_0 \cdot w_1^{-\eta_1}$ and $S(w_1) = S_0 \cdot w_1^{e_1}$ so that $S_0 \cdot w_1^{*e_1} = D_0 \cdot w_1^{*-\eta_1}$ and $S_0 \cdot \underline{w}^{e_1} = D_0 \cdot \bar{w}^{-\eta_1}$. This implies that $\underline{w} = w_1^* \cdot (w_1^*/\bar{w})^{\eta_1/e_1}$, and hence:

$$\frac{\bar{w} - \underline{w}}{\bar{w}} = 1 - \left(\frac{w_1^*}{\bar{w}}\right)^{1+\frac{\eta_1}{e_1}}.$$

This equation shows that for a given wedge $(\bar{w} - \underline{w})/\bar{w}$, the minimum wage markup $\bar{w}/w_1^*$ is *increasing* in e_1 and *decreasing* in η_1. Formula (5) can be rewritten as:

$$\frac{\bar{w}}{w_1^*} = \left(1 - \frac{g_1 - g_2}{g_0^e \cdot \eta_1}\right)^{-\frac{e_1}{e_1+\eta_1}} \simeq 1 + \frac{e_1}{e_1 + \eta_1} \cdot \frac{g_1 - g_2}{g_0^e \cdot \eta_1}, \tag{6}$$

where the approximation holds in the case of a small minimum wage (i.e., when $(g_2 - g_1)/(g_0^e \cdot \eta_1)$ is small). This formula has the advantage of providing an explicit formula for the minimum wage mark-up above the market wage w_1^* expressed in terms of supply and demand elasticities and the social welfare weights. It shows that the optimum minimum wage $\bar{w}$ is decreasing in the supply elasticity e_1. The intuition can be easily understood from Figure 2. A higher supply elasticity, implies a flatter supply curve, and hence lower costs from involuntary unemployment. If the supply elasticity is high, then a small change in w_1 has large effects on supply, implying that many workers derive a very small surplus from working and hence do not lose much when they are forced out of work by a minimum wage. This result is very important because, as is well known, redistribution through taxes and transfers is hampered by a high supply elasticity. As we will see later, situations with high supply elasticities for low skilled workers make it particularly valuable to use the minimum wage even in the presence of optimal taxes. Conversely, when the supply elasticity is low, redistribution through the minimum wage is costly while redistribution through taxes and transfers is efficient.[14]

Formula (6) shows that there are two channels through which a higher demand elasticity η_1 reduces the optimal minimum wage. The first channel is the standard *unemployment level effect* developed when discussing (5) that higher demand elasticity creates a larger unemployment

[14]This shows also that the case with no supply elasticity upon which the labor literature has focused is not only unfavorable to the minimum wage but also a case where the minimum wage would be clearly dominated by tax and transfer policy.

response to the minimum wage. The second channel is an *unemployment cost* effect which works through the link between the wedge $(\bar{w}-\underline{w})/\bar{w}$ and the minimum wage markup $\bar{w}/w_1^*$. A higher demand elasticity implies that a given minimum wage markup is associated with a larger wedge, hence higher unemployment costs for the marginal worker. The distinction between those two channels is important because we will see that the first classical unemployment level effect disappears with optimal taxes and transfers but the unemployment cost effect remains.

4 Optimal Minimum Wage with Taxes and Transfers

4.1 The Basic Two-Skill Model

Let us now assume that the government uses taxes and transfers. In our simple two-skill model, we assume that the government can observe job outcomes (not working, work in sector 1 paying w_1, or work in sector 2 paying w_2) but does not observe costs of work. Therefore, the government can condition tax and transfers only on those observable work outcomes. Let us denote by T_i the tax (or transfer if $T_i < 0$) on occupation i. We denote by $c_i = w_i - T_i$ the disposable income in occupation $i = 0, 1, 2$. This is a fully general nonlinear income tax on earnings.

As in our previous model without taxes, an individual with skill $i = 1, 2$ who decides to work earns w_i but increases his disposable by $c_i - c_0$. Hence we can naturally define a tax rate τ_i on skill i workers: $1-\tau_i = (c_i - c_0)/w_i$. An individual of skill $i = 1, 2$ and with costs of work θ works if and only if $\theta \leq c_i - c_0 = (1 - \tau_i)w_i$. Hence, the aggregate labor supply functions for $i = 1, 2$ are:

$$h_i = h_i^0 \cdot P_i((1 - \tau_i)w_i) = h_i^0 \cdot P_i(c_i - c_0). \tag{7}$$

As above, we denote by e_i the elasticity of labor supply with respect to the net-of-tax wage rate $w_i(1 - \tau_i) = c_i - c_0$:

$$e_i = \frac{(1 - \tau_i)w_i}{h_i} \frac{\partial h_i}{\partial(1 - \tau_i)w_i} = \frac{(1 - \tau_i)w_i \cdot p_i((1 - \tau_i)w_i)}{P_i((1 - \tau_i)w_i)},$$

The demand side of the economy is unchanged. For given parameters c_0, τ_1, τ_2 defining a tax and transfer system, the four equations (1) and (7) for $i = 1, 2$ define the competitive equilibrium $(h_1^*, h_2^*, w_1^*, w_2^*)$.

Assuming no exogenous spending requirement, the government budget constraint can be written as:

$$h_0 c_0 + h_1 c_1 + h_2 c_2 \leq h_1 w_1 + h_2 w_2, \tag{8}$$

which can be rewritten as $c_0 = h_1 \tau_1 w_1 + h_2 \tau_2 w_2$. None of our results would be changed if we assumed a positive exogenous spending requirement for the government.

- **Minimum Wage Desirability with Fixed Tax Rates**

Let us first analyze how our previous analysis on the desirability of the minimum wage is affected in the presence of taxes and transfers assuming that τ_1, τ_2 are exogenously fixed and that the transfer to non-workers c_0 adjusts automatically to meet the government budget constraint when a small minimum wage $\bar{w} = w_1^* + d\bar{w}$ is introduced. We assume that the minimum wage applies to wages before taxes and transfers.[15] This assumption does not affect the desirability of a minimum wage and is the most convenient convention. Let us define social marginal welfare weights at each occupation level as:

$$g_i = \frac{\int G'(c_i - \theta) dP_i(\theta)}{\lambda \cdot P_i},$$

where the integration takes places over all individuals in occupation $i = 1, 2$. We define $g_0 = G'(c_0)/\lambda$. λ is a normalization parameter defined such that:

$$h_0 g_0 + h_1 g_1 + h_2 g_2 = 1$$

We will show below that λ is the multiplier of the government budget constraint when the government optimizes taxes and transfers.

Proposition 2 *With fixed tax rates τ_1, τ_2, under the efficient rationing assumption 1 and assuming $e_1 > 0$ and $\eta_1 < \infty$, introducing a minimum wage is desirable iff*

$$g_1 \cdot (1 - \tau_1) - g_2 \cdot (1 - \tau_2) + \tau_1 - \tau_2 - \tau_2 \cdot e_2 - \tau_1 \cdot \eta_1 > 0. \tag{9}$$

When $\tau_1 = \tau_2 = 0$, equation (9) boils down to $g_1 - g_2 > 0$ which is fully consistent with proposition 1. Equation (9) shows with taxes and transfers, introducing a minimum wage

[15] In practice, the legal minimum wage applies to wages net of employer payroll taxes but before employee payroll taxes, income taxes, and transfers. $\bar{w}$ should be interpreted as the minimum wage gross of all employer taxes.

creates four fiscal effects that need to be taken into account in the welfare analysis: first, transferring \$1 pre-tax from high skilled workers to low skilled workers through the minimum wage implies a \$ $(1-\tau_1)$ post tax transfer to low skilled workers and a \$ $(1-\tau_1)$ post tax loss to high skilled workers, hence the factors $(1-\tau_i)$ multiplying g_1 and g_2 in 9. Second and related, such a \$1 transfer creates a direct net fiscal effect $\tau_1 - \tau_2$. Third, the reduction in w_2 leads to a supply effect which further reduces taxes paid by the high skilled by $e_2\tau_2$ per dollar transferred. Finally, involuntary unemployment also creates a tax loss equal to $-\tau_1 \cdot \eta_1$ per dollar transferred.[16]

- **Optimal Tax Formulas with no Minimum Wage**

As established by Diamond and Mirrlees (1971), one can ignore demand effects and assume that wage rates w_i are constant when deriving optimal tax rates.[17] Let us denote by λ the multiplier of the budget constraint. Let us denote by g_i the average social marginal welfare weight in group $i = 0, 1, 2$. By definition $g_0 = G'(c_0)/\lambda$ and $g_i = \int_0^{c_i - c_0} G'(c_0 - c_i - \theta)p_i(\theta)d\theta/(\lambda \cdot P_i)$ for $i = 1, 2$. Following Saez (2002), we have the following conditions at the optimum:

$$h_0 \cdot g_0 + h_1 \cdot g_1 + h_2 \cdot g_2 = 1, \tag{10}$$

$$\frac{\tau_i}{1-\tau_i} = \frac{1-g_i}{e_i}. \tag{11}$$

for $i = 1, 2$. Those equations are immediately derived from the first order conditions of the government maximization with respect to c_0, and c_i, $i = 1, 2$ respectively. Equation (10) implies that the average of marginal welfare weights across the three groups $i = 0, 1, 2$ is one. Indeed, the value of distributing one dollar to everybody is exactly the average marginal social weight and the cost of distributing one dollar in terms of revenue lost is also one dollar as we have assumed away income effects.[18]

Equation (11) is a simple inverse elasticity rule. If $g_1 > 1$, then the optimal tax rate on low skilled workers should be negative so that in-work benefits such as the EITC are optimal.

[16] Note that when low skilled work is subsidized ($\tau_1 < 0$), then the unemployment created by a small minimum wage creates a positive fiscal externality which proportional to the the demand elasticity η_1. In such a situation, introducing a minimum wage would actually be desirable even without redistributive tastes ($g_1 = g_2 = 1$) if $-\tau_1 \cdot \eta_1 > \tau_2 \cdot e_2$.

[17] Piketty (1997) and Saez (2004) have shown that the occupational model we consider inherits this important property of the Diamond and Mirrlees (1971) model.

[18] With income effects, distributing one dollar reduces labor supply which in general reduces tax revenue and hence the average marginal social weight is larger than one. See Saez (2002) for a more detailed discussion. Our key results would not be affected by the presence of income effects.

This was the key result emphasized by Diamond (1980), Saez (2002), Laroque (2005), Chon and Laroque (2005, 2006): an EITC type transfer for low wage workers can be optimal in a situation where individuals respond only along the extensive margin. This result can be understood from Figure 4. Starting from an allocation (c_0, c_1, c_2), and increasing c_1 by dc_1 leads to a positive direct welfare effect $h_1 g_1 dc_1$, a mechanical loss in tax revenue $-h_1 dc_1$, and a behavioral response increasing work $dh_1 = e_1 h_1 (c_1 - c_0) dc_1 > 0$ and creating a fiscal effect equal to $\tau_1 w_1 dh_1$. If $g_1 > 1$, then, at the optimum, the behavioral response fiscal effect has to be negative implying that $\tau_1 < 0$ at the optimum.

Figure 4b illustrates the important point that the optimal tax/transfer derivation is unchanged when w_1, w_2 are endogenous. In that case, the small reform dc_1 leads to changes in h_1 and hence to changes dw_1 and dw_2 through demand side effects. However, assuming that c_2 and $c_1 + dc_1$ are kept unchanged, the effect of dw_1 and dw_2 is fiscally neutral because $h_1 dw_1 + h_2 dw_2 = 0$ through the no-profit condition.

Let us denote by (w_i^T, c_i^T) the tax/transfer optimum with no minimum wage.

- **Desirability of a Minimum Wage**

As illustrated on Figure 5, starting from the tax/transfer optimum (w_i^T, c_i^T), let us introduce a minimum wage set at $\bar{w} = w_1^T$. Such a minimum wage is just binding and has no direct impact on the allocation. Let us now increase c_1 by dc_1 while keeping c_0 and c_2 constant. As we showed above, such a change provides incentives for some low skilled individuals to start working. However, as we showed in Figure 4b, such a labor supply response would reduce w_1 through demand side effects. However, in the presence of a minimum wage set at w_1^T, w_1 cannot fall, which implies that those individuals willing to start working cannot work and actually shift from voluntary to involuntary unemployment. The assumption of efficient rationing is key here as these are precisely the individuals with the lowest surplus from working.

Given that the labor supply channel is effectively shut down by the minimum wage, the dc_1 change is like a lumpsum tax reform and its net welfare effect is simply $[g_1 - 1] h_1 dc_1$. This implies that if $g_1 > 1$, then this reform is desirable which implies that introducing a minimum wage allows to improve upon the tax/transfer optimum allocation. Therefore, we can state:

Proposition 3 *Under the efficient rationing assumption 1, assuming $e_1 > 0$ and $\eta_1 < \infty$, and that the government uses taxes and transfers optimally, if $g_1 > 1$ at the optimum tax* *allocation (with no minimum wage) then introducing a minimum wage is desirable.*

Proposition 3 is fairly similar to our initial proposition with no taxes and transfers. The condition $g_1 > 1$ is somewhat weaker than $g_1 > g_2$ (as the g_i average to one and hence $g_2 < 1$). However, if the government has redistributive tastes, then $g_1 > 1$ is a weak condition especially given that the low skilled sector can be chosen to represent very low income workers. In that case, $g_1 > 1$ breaks down only if the weight g_0 on the non-workers is so high as to make the weights on workers, no matter how low their wage, below 1. This happens in the case of the extreme Rawlsian welfare criteria. The condition $g_1 > 1$ means that the government values redistribution toward low wage workers. In that case, the minimum wage helps the government redistribute more to low wage workers by shutting down the labor supply behavioral response channel.

Proposition 3 can also be derived easily from Proposition 2 by plugging the optimal tax rates from equations (11). In that case, equation (9) boils down to $-\tau_1 \cdot (e_1 + \eta_1) > 0$ which is indeed equivalent to $g_1 > 1$.

This result is line with the theory of optimum quantity controls developed by Guesnerie (1981) and Guesnerie and Roberts (1984) showing that, in an optimum tax model, introducing a quantity control on subsidized goods is desirable. In our model, a minimum wage is an indirect way for the government to introduce rationing on low skilled work.[19] As we will see in Section 3.2, this result generalizes easily to a model general model with many skills and fully general labor supply responses functions.

- **Optimum Tax/Transfer and Minimum Wage Allocation**

Formally, the government chooses $\bar{w}, c_0, c_1, c_2$ to maximize

$$SW = (1-h_1-h_2)G(c_0) + h_1^0 \cdot \int_0^{\underline{w}(1-\tau_1)} G(c_1-\theta)p_1(\theta)d\theta + h_2^0 \cdot \int_0^{c_2-c_0} G(c_2-\theta)p_2(\theta)d\theta. \quad (12)$$

subject to its budget constraint (with multiplier λ), the supply function constraint (if the minimum wage does not bind), and the fact that wage rates are competitively set. As above, $\underline{w}$ is defined as the reservation wage of the marginal worker: $h_1^0 \cdot P_1(\underline{w}(1-\tau_1)) = D_1(\bar{w})$ where $D_1(\bar{w})$ is the demand for low skilled labor for a given minimum wage $\bar{w}$. Formulation of the second term in (12) incorporates the efficient rationing assumption.

[19]Guesnerie and Roberts (1987) proposed an analysis of optimal minimum wage. However, the model they considered was not directly related to their quota theory as we discuss below.

As above, the first order condition with respect to c_0 implies that λ is such that $h_0 g_0 + h_1 g_1 + h_2 g_2 = 1$.

With a binding minimum wage, as illustrated on Figure 6, increasing c_1 is a lumpsum transfer. Therefore, the government will increase c_1 up to point where $g_1 = 1$. Therefore, the first order condition for the optimum c_1 leads to:

Corollary 1 *Under the efficient rationing assumption 1 and if $g_1 > 1$ at the tax optimum with no minimum wage (i.e., when a minimum wage is desirable), then, at the joint tax/transfer and minimum wage optimum, we have $g_1 = 1$.*

This corollary shows that the minimum wage is a powerful tool to increase redistribution to low wage workers as it can bring their average social marginal welfare weight up to the average among all individuals, effectively reaching the point where the government no longer values further redistribution toward them.

The first order condition with respect to c_2 leads to the standard formula (11): $\tau_2/(1-\tau_2) = (1-g_2)/e_2$ as the minimum wage does not impact the trade-off for the choice of c_2.

Finally, there is a first order condition for the optimal choice of $\bar{w}$ that is illustrated on Figure 7. Increasing $\bar{w}$ by $d\bar{w}$ and keeping c_0, c_1, c_2 constant leads to an increase in involuntary unemployment: $dh_1 < 0$. Such involuntary unemployment leads to a (negative) welfare effect on those individuals equal to $dh_1[G(c_0 + (\bar{w} - \underline{w})(1-\tau_1)) - G(c_0)]/\lambda < 0$ and a fiscal effect equal to $dh_1 \cdot \tau_1 \cdot \bar{w}$, which leads to the following first order condition:

$$-\tau_1 \cdot \bar{w} = \frac{G(c_0 + (\bar{w} - \underline{w})(1-\tau_1)) - G(c_0)}{\lambda}. \tag{13}$$

As we did in Section 2, we can introduce the social marginal weight on earnings losses due to (marginal) involuntary unemployment: $g_0^e = [G(c_0 + (\bar{w} - \underline{w})(1-\tau_1)) - G(c_0)]/[\lambda(\bar{w} - \underline{w})(1-\tau_1)]$ in order to rewrite (13) as:

$$\frac{\bar{w} - \underline{w}}{\bar{w}} = -\frac{\tau_1}{1-\tau_1} \cdot \frac{1}{g_0^e} > 0. \tag{14}$$

If $g_1 > 1$ at the tax optimum with no minimum wage, then a minimum wage is desirable and as a result, equation (FOCbarw2) implies that the tax rate on low skilled workers τ_1 should be negative at the joint tax/minimum wage optimum. Consequently, we can state the following important result:

Proposition 4 *In our model with extensive labor supply responses, a binding minimum wage associated with a positive tax rate at the bottom* $\tau_1 > 0$ *is second-best Pareto inefficient. This latter result remains true even when rationing is not efficient.*

Proposition 4 is illustrated on Figure 8. Suppose that the minimum wage binds and that $\tau_1 > 0$. This is a situation most OECD countries face, especially in continental Europe where traditional transfer programs combined with substantial payroll tax rates create very large tax rates on low skilled work. Suppose that the government reduces the minimum wage by $d\bar{w} < 0$ while keeping c_0, c_1, c_2 constant.[20] Reducing the minimum wage leads to a positive employment effect $dh_1 > 0$ as involuntary unemployment is reduced which improves the welfare of the newly employed workers and also increases tax revenue as $\tau_1 > 0$. It is important to note that the increase in welfare due to the reduction in unemployment remains a-fortiori true if rationing is not efficient. The increase $dh_1 > 0$ also leads to a change $dw_2 > 0$. Because $h_1 d\bar{w} + h_2 dw_2 = 0$, the mechanical fiscal effect of $d\bar{w}$ and dw_2 while keeping c_1 and c_2 constant is zero.[21] Therefore, this reform is a (second-best) Pareto improvement which immediately suggests a double dividend policy reform in countries such as France with high minimum wages combined with high tax rates at the bottom.[22]

Quantitatively, τ_1 is primarily determined to meet the condition $g_1 = 1$. Then, the optimal minimum wage wedge $(\bar{w} - \underline{w})/\bar{w}$ is determined by equation (14) and is increasing in the size of the absolute subsidy $|\tau_1|$ and decreasing in the social weight on unemployment earnings losses g_0^e. As we discussed in Section 2, we can define the implicit market wage rate w_1 as the wage rate that would prevail under the same tax rates τ_1, τ_0 but with no minimum wage. In that case, and assuming constant elasticity of supply and demand, we showed that the minimum wage markup over the market wage rate $\bar{w}/w_1$, for a given minimum wage wedge $(\bar{w} - \underline{w})/\bar{w}$ was increasing in e_1 and decreasing in η_1, suggesting that our previous results that the optimum minimum wage increases with e_1 and decreases with η_1 carry over to the case with optimal taxes. It is important to note that a high demand elasticity leads to a smaller minimum wage

[20] In practice, this could be achieved by cutting the employer payroll taxes for low income workers, which lowers the (gross) minimum wage without affecting the net minimum wage after taxes and transfers.

[21] In practice, keeping c_2 constant while w_2 increases means that it will be necessary to increase the tax rate on higher skilled workers.

[22] Actually, such a double dividend policy reform was started in France in the early 1990s when the employer payroll tax on low income workers was reduced (see Crépon and Desplatz, 2002 for an empirical analysis). However, if our theory is close approximation of the real world, this policy should be pushed further.

not because this creates more unemployment but rather because a large demand elasticity makes unemployment more costly by increasing the wedge $(\bar{w} - \underline{w})/\bar{w}$. As we discussed in Section 2, the unemployment level channel vanishes with optimal taxes but the unemployment cost channel remains.

The result that the optimum minimum wage follows an inverted U-shape pattern with the strength of redistributive tastes also carries over to this case. As we have seen, extreme redistributive tastes imply that $g_1 < 1$ and hence no minimum wage is desirable and conversely, no redistributive tastes imply that $g_0 = g_1 = g_2 = 1$, a situation where no minimum wage is desirable either.

- **Discussion: Wages vs. Earnings**

In practice, minimum wages are based on hourly wage rates. There are two reasons why wage rates might not be a good indicator of earnings or economic welfare.

First, minimum wage workers might belong to families with higher incomes such as in the case of a person with a high income spouse or a teenager with high income parents. Johnson and Browning (1983) and Burkhauser, Couch, and Glenn (1996) propose an empirical analysis of this issue in the United States. This is straightforward to incorporate in the analysis by simply altering the social marginal welfare weights. Our optimal formulas continue to hold in that context. For example, if all minimum wage workers were teenagers from middle class families, g_1 could possibly be lower than g_2 and no minimum wage would be desirable.

Second, we have made the assumption that involuntary unemployment is not shared and falls in a binary fashion on workers. This assumption is reasonable if low income workers who become unemployed do not have buffer stock savings, or do not have access to credit markets or other resources to smooth consumption overtime. If unemployed workers can smooth consumption and unemployment spells are relatively short, a better assumption would be that unemployment is shared from a longer term perspective. In that case, the utility loss due to unemployment is lower. This can be incorporated in the analysis by replacing the weight g_0^e by another weight g_0^s that is actually much closer to g_1. This obviously leads to a higher optimum minimum wage but does not change the fundamental formulas.

5 Extensions

5.1 Uniform Rationing

As discussed above, our previous results are derived under the key assumption of efficient rationing, which is the situation the most favorable to the minimum wage. It is therefore useful to explore briefly how results change if we drop this assumption and adopt instead a "uniform rationing" assumption whereby unemployment is distributed across workers independently of surplus. Uniform rationing can be seen as the opposite polar assumption to efficient rationing. Empirically, we expect rationing to be in between those two extreme cases with suggestive evidence discussed above that rationing might be closer to efficient than uniform at least in the US labor market.

- **Case with no Taxes**

In the case of uniform rationing, the government chooses $\bar{w}$ to maximize:

$$SW = (1 - D_1(\bar{w}) - h_2)G(0) + D_1(\bar{w}) \cdot \int_0^{\bar{w}} G(\bar{w} - \theta)\frac{p_1(\theta)}{P_1(\bar{w})} d\theta + h_2^0 \cdot \int_0^{w_2} G(w_2 - \theta)p_2(\theta)d\theta. \quad (15)$$

The second term in equation (15) reflects the facts that all workers with work costs $\theta \in (0, \bar{w})$ have the same probability of being employed but that the total number of low skilled workers is given by the demand function $D_1(\bar{w})$.

Figure 3 illustrates the trade-off involved in increasing the minimum wage $\bar{w}$ by $d\bar{w}$ under the "uniform rationing" scenario. The redistributive value of introducing a small minimum wage $d\bar{w}$ remains the same: $T = [g_1 - g_2]h_1 d\bar{w}$. As shown on Figure 3, the minimum wage reduces employment through a demand effect by $dh_1 = -\eta_1 h_1 d\bar{w}/\bar{w}$. However, the minimum wage will induce workers with cost of work $\theta \in (\bar{w}, \bar{w} + d\bar{w})$ to look for a job as well. There are $e_1 h_1^S d\bar{w}/\bar{w}$ such workers where $h_1^S = h_0^1 P_1(\bar{w})$. Under efficient rationing, those marginal workers would stay out of work. Under uniform rationing, however, a fraction h_1/h_1^S of those new workers will join the labor force and will displace some other workers as unemployment is distributed uniformly. That excess labor supply creates involuntary unemployment. As involuntary unemployment is distributed uniformly across all low skilled workers, the average welfare cost per displaced worker is $\int_0^{\bar{w}} [G(\bar{w} - \theta) - G(0)]p_1(\theta)d\theta/P_1(\bar{w})$. The number of displaced workers is $h_1(e_1 + \eta_1)d\bar{w}/\bar{w}$. Thus, the welfare loss due to involuntary unemployment is equal to $U = -h_1(d\bar{w}/\bar{w})(e_1 + \eta_1)\int_0^{\bar{w}} [G(\bar{w} - \theta) - G(0)]p_1(\theta)d\theta/P_1(\bar{w})$. At the optimum, we have $U + T = 0$ which implies

$$\int_0^{\bar{w}} \frac{[G(\bar{w} - \theta) - G(0)]p_1(\theta)d\theta}{\bar{w}P_1(\bar{w})} \cdot (e_1 + \eta_1) = g_1 - g_2. \tag{16}$$

If at $\bar{w} = w_1^*$, the left-hand-side is smaller than the right-hand-side of (16), then a minimum wage is desirable (and conversely). The key point is that a minimum wage is no longer necessarily desirable under "uniform rationing".

We can introduce a welfare weight on employment losses defined as $g_0^u = \int_0^{\bar{w}}[G(\bar{w} - \theta) - G(0)]p_1(\theta)d\theta / \int_0^{\bar{w}}(\bar{w} - \theta)p_1(\theta)d\theta$. If we assume that the supply elasticity e_1 is constant, then $p_1(\theta) = C \cdot \theta^{e_1}$ and hence $\int_0^{\bar{w}}(\bar{w} - \theta)p_1(\theta)d\theta/P_1(\bar{w}) = \bar{w}/(1 + e_1)$. In this case, we can rewrite (16) as follows:

$$\frac{e_1 + \eta_1}{1 + e_1} = \frac{g_1 - g_2}{g_0^u}. \tag{17}$$

This equation is an implicit formula for the optimum minimum wage. Presumably, the welfare weight ratio $(g_1 - g_2)/g_0^u$ is decreasing with $\bar{w}$. Formula (17) implies that the minimum wage should be increased up to the point where the welfare weight ratio is equal to the elasticity ratio $(e_1 + \eta_1)/(1 + e_1)$. Obviously, if at $\bar{w} = w_1^*$, the welfare weight ratio is already below the elasticity ratio, then no minimum wage is desirable. Note that the elasticity ratio is increasing in η_1. In most cases, we would expect that $g_0^u \geq g_1$.[23] As a result, equation (17) implies that the right-hand-side is less than one, and hence a minimum wage can be desirable only if $\eta_1 < 1$. The optimum minimum wage is decreasing in η_1.

Interestingly, when $\eta_1 < 1$, the elasticity ratio increases with e_1. This implies that the optimum minimum wage is decreasing in e_1. This stands in contrast to our results with efficient rationing and can be understood as follows. A large supply elasticity makes unemployment less costly as workers have lower surplus from working on average but a large supply elasticity induces more formerly out of work individuals to start looking for jobs and displace workers with higher surplus which is inefficient. When $\eta_1 < 1$, the latter effect is stronger than the former effect explaining why the minimum wage decreases with e_1.[24]

[23] For example, this holds for constant supply elasticity e_1 and constant risk aversion functions $G(.)$. However, it is possible to find functions $G(.)$ and $p(.)$ where this does not hold.

[24] Note also that, with uniform rationing, and if workers can smooth consumption across unemployment spells, then we have $g_0^u = g_1$. The standard result about the pivotal $\eta_1 = 1$ can be seen as a particular case of (17) when $e_1 = 0$ (no supply elasticity), $g_2 = 0$ (no value assigned to high skilled workers), and $g_0^u = g_1$ (unemployment spells are shared and consumption is smoothed).

Empirically, we would expect rationing to be in between efficient rationing and uniform rationing. It is very easy to extend the model to a mixed situation where a fraction δ of unemployment strikes uniformly while a fraction $1-\delta$ of the unemployment is efficiently allocated. In that case, the formula for the optimum minimum wage is a straight average of (5) and (17), namely $g_1 - g_2 = (1-\delta)g_0^e \eta_1(\bar{w} - \underline{w})/\bar{w} + \delta g_0^u(\eta_1 + e_1)/(\eta_1 + e_1)$. This shows that our efficient rationing results are *robust* to the introduction of a little bit of uniform rationing.

- **Case with Optimal Taxes**

In that case, the government chooses, c_0, c_1, c_2, and $\bar{w}$ to maximize:

$$SW = (1-D_1(\bar{w})-h_2^0 P_2(c_2-c_0))G(c_0)+D_1(\bar{w})\cdot\int_0^{c_1-c_0} G(c_1-\theta)\frac{p_1(\theta)}{P_1(c_1-c_0)}d\theta+h_2^0\cdot\int_0^{c_2-c_0} G(c_2-\theta)p_2(\theta)d\theta, \tag{18}$$

subject to the standard budget constraint and the fact that demand for labor is competitively set. The second term in equation (18) reflects the facts that all workers with work costs $\theta \in (0, c_1 - c_0)$ have the same probability of being employed but that the total number of low skilled workers is given by the demand function $D_1(\bar{w})$. The first order condition with respect to c_0 (keeping $c_1 - c_0, c_2 - c_0, \bar{w}$ constant) implies the standard result $h_0 g_0 + h_1 g_1 + h_2 g_2 = 1$. The first order condition with respect to c_2 leads to the standard optimal tax formula for τ_2, namely $\tau_2/(1-\tau_2) = (1-g_2)/e_2$.

The first order condition with respect to c_1 leads to:

$$\frac{g_1 - 1}{e_1} = g_0^u \cdot \int_0^{c_1-c_0} \left(1 - \frac{\theta}{c_1 - c_0}\right)\frac{p_1(\theta)}{P_1(c_1-c_0)}d\theta, \tag{19}$$

where $g_0^u = \int_0^{c_1-c_0}[G(c_1-\theta) - G(c_0)]p_1(\theta)d\theta/(\lambda \cdot \int_0^{c_1-c_0}(c_1 - c_0 - \theta)p_1(\theta)d\theta)$ is the welfare weight on unemployment losses.

The first order condition with respect to $\bar{w}$ leads to:

$$-\frac{\tau_1}{1-\tau_1} = g_0^u \cdot \int_0^{c_1-c_0} \left(1 - \frac{\theta}{c_1 - c_0}\right)\frac{p_1(\theta)}{P_1(c_1-c_0)}d\theta. \tag{20}$$

Therefore and strikingly, combining those two first order conditions, we find that the optimal tax formula for τ_1 *in the presence of the optimal minimum wage* is the same as with no minimum wage, namely $\tau_1/(1-\tau_1) = (1-g_1)/e_1$. Intuitively and following the derivation from Figure 4b, this can be understood as follows. Suppose c_1 is increased by dc_1 and at the same time, the minimum wage $\bar{w}$ is reduced by $d\bar{w}$ such that $dc_1 \cdot p_1/P_1 = d\bar{w} \cdot D_1'(\bar{w})/D_1$. In that case, a fraction D_1/P_1 of those $p_1 dc_1$ workers willing to join the labor force because of dc_1 can do so and hence the fiscal effect of the reform is $(T_1 - T_0) p_1 dc_1 \cdot D_1/P_1 = D_1 dc_1 \cdot e_1 \cdot \tau_1/(1-\tau_1)$ and hence the standard formula goes through. We can then obtain the following proposition.

Proposition 5 *With optimal taxes and transfers and uniform rationing, if the welfare weight on unemployment losses is larger than the welfare weight on low skilled workers ($g_0^u \geq g_1$) and the supply elasticity e_1 is constant, then a minimum wage is not desirable.*

Proof: Under the assumption of a constant e_1 and if the minimum wage binds, the integral term in the right-hand-side of (19) is equal to $1/(1 + e_1)$ and hence (19) can be rewritten as $(g_1 - 1)/e_1 = g_0^u/(1 + e_1)$. However, $(g_1 - 1)/e_1 < g_1/(1 + e_1) \leq g_0^u/(1 + e_1)$, where the first inequality follows from that fact that $g_1 < 1 + e_1$ (as $\tau_1 = (1 - g_1)/(1 - g_1 + e_1)$)[25] and the second inequality from our assumption that $g_1 \leq g_0^u$. This creates a contradiction showing that the minimum wage cannot be binding.

Intuitively, one can understand those results as follows. With efficient rationing, the minimum wage effectively reveals the marginal workers to the government. Because costs of work are not observable, this is valuable as it allows the government to sort workers into a more (socially albeit not privately) efficient set of occupations. Therefore, the minimum wage is desirable. In contrast, with uniform rationing, the minimum wage does not reveal anything about costs of work (as unemployment strikes randomly). As a result, the minimum wage just creates (privately) inefficient sorting across occupations but without revealing anything of value to the government. It is not surprising that the minimum would not be desirable in such a context. Furthermore, because unemployment is random, the trade-offs involved when setting the optimal tax system are effectively unchanged explaining why the optimal tax formulas do not change.[26]

Therefore, and perhaps not surprisingly, the desirability of a minimum wage hinges crucially on whether rationing is efficient or not. This makes it all the more important to make empirical progress on this question. As in the case with no taxes, it is easy to extend the model to a mixed situation where a fraction δ of unemployment strikes uniformly while a fraction $1 - \delta$ of the unemployment is efficiently allocated. In particular, a minimum is desirable if and only

[25]If $g_1 > 1 + e_1$, then reducing τ_1 is strictly desirable which cannot happen at the optimum.

[26]We conjecture that this result can be generalized to many skills and any labor supply functions.

if $g_1 > 1 + \delta g_0^u e_1/(1+e_1)$ at the tax optimum with no minimum wage. When a minimum wage is desirable, at the optimum we have, $g_1 = 1 + \delta g_0^u e_1/(1+e_1)$ and $-\tau_1/(1-\tau_1) = \delta g_0^u/(1+e_1) + (1-\delta) g_0^e(\bar{w} - \underline{w})/\bar{w}$. This shows again that our results under efficient rationing are robust to the introduction of a little bit of uniform rationing (moderate δ).

5.2 General Labor Supply Function

In this section we consider a general model with I occupations (instead of 2) and a general production function.[27] More importantly, the model will allow for any labor supply responses, instead of only considering the extensive margin as in the previous section.

- **Model and Optimal Taxation**

The model we use is the general occupation model described in the appendix of Saez (2002) and in Saez (2004). There are $I+1$ occupations, paying wages $w_0 = 0, w_1, .., w_I$. Occupation 0 denotes unemployment. We assume that in equilibrium, occupations are ordered so that $0 < w_1 < .. < w_I$. For simplicity, we assume away income effects but we consider a fully general supply side model.[28] Each individual is therefore characterized by a cost parameter $\theta = (\theta_0 = 0, \theta_1, .., \theta_I)$ which describes the labor supply cost for the individual to work in each occupation $i = 0, 1, .., I$. By assumption, being out of work is costless. We assume that θ is distributed according to a measure $\nu(\theta)$ on Θ, with total population normalized to one.

The government can apply a general income tax and transfer system $T = (T_0, .., T_I)$. We denote by $c_i = w_i - T_i$ the disposable income (after taxes and transfers) in occupation i. An individual with cost θ picks the occupation i which maximizes $c_j - \theta_j$ for $j = 0, .., I$. Hence, the set Θ is partitioned into $I+1$ subsets $\Theta_0, .., \Theta_I$ so that individuals with $\theta \in \Theta_i$ choose occupation i. We denote by $h_i = \nu(\Theta_i)$ the fraction of individuals in occupation i. Those supply functions are functions of $c = (c_0, .., c_I)$ and hence denoted by $h_i(c_0, .., c_I)$. We assume that θ is distributed smoothly across individuals so that the supply functions h_i are continuously differentiable. This is a fully general supply model with no income effects. Our no income effect assumption implies that for any R, $h_i(c_0 + R, .., c_I + R) = h_i(c_0, .., c_I)$ so that $\sum_{j=0}^{I} \partial h_i/\partial c_j = 0$.

[27]Introducing a capital input would also be possible as long as we assume that returns on capital can be taxed at a specific rate τ_K.

[28]The key results on the desirability of the minimum wage easily generalize to the case with income effects.

The participation model from our previous section is a special case. Similarly, the intensive labor supply of Mirrlees (1971) can be represented in this discrete model by assuming that individuals of "type i" can work in job $i-1$ at no cost or work in job i at cost $\theta_i > 0$ (see Saez 2002 for details).

Abstracting first from the minimum wage, the government chooses $c = (c_0, .., c_I)$ in order to maximize: $SW = \int_{\theta \in \Theta} G(c_i - \theta_i) d\nu(\theta)$ subject to the budget constraint: $\sum_{j=0}^{I} (w_j - c_j) \cdot h_j(c) \geq 0$. $G(.)$ is increasing and concave and where index i inside in integral for SW denotes the utility maximizing job choice of individual θ. We denote again by λ the multiplier of the budget constraint.

Assuming first that the w_i are fixed, the first order condition with respect to c_i is simply:

$$(1 - g_i) h_i = \sum_{j=0}^{I} T_j \cdot \frac{\partial h_j}{\partial c_i}, \tag{21}$$

where g_i is the average social marginal welfare weight in occupation i, defined as $g_i = \int_{\theta \in \Theta_i} G'(c_i - \theta_i) d\nu(\theta) / (\lambda \cdot h_i)$.

The derivation is straightforward once one recognizes that the welfare effect of an small increase dc_i due to switching jobs behavioral responses is zero (because of a standard envelope theorem argument). The no income effects assumption implies that $\sum_{j=0}^{I} g_i \cdot h_i = 1$. This can be obtained by increasing every c_i by dc uniformly. This generates no behavioral responses and hence the fiscal cost dc must be equal to the welfare gain $dc \cdot \sum_j h_j g_j$. This implies that the average of g_i is one. It is natural to assume that $g_0 > g_1 > .. > g_I$. Thus low pay occupations will likely have $g_i > 1$: Absent behavioral responses, the government would like to redistribute toward those groups.

Let us assume now that w_i are endogenous and defined as the marginal product of a general production function $F(h_1, .., h_I)$. We assume that F has constant returns to scale so that there are no profits. It turns out that formula (21) remains valid in that case as well. This is a well known result in optimal taxation originally demonstrated by Diamond and Mirrlees (1971). The intuition can easily be seen as follows: Increasing c_i by dc_i generates changes in all wages $dw_1, .., dw_I$ through general equilibrium demand effects. Keeping each c_j (for $j \neq i$) constant, the fiscal cost of such changes $dw_1, .., dw_I$ is $h_1 dw_1 + .. + h_I dw_I$. This expression is zero because there are no profits in equilibrium: $0 = F(h_1, .., h_I) - (w_1 h_1 + .. + w_I h_I)$, implies that $h_1 dw_1 + .. + h_I dw_I = 0$ (as $\partial F / \partial h_i = w_i$).

• **Desirability of Minimum Wage Rationing**

We can generalize proposition 3 as follows:

Proposition 6 *If the efficient rationing Assumption 1 holds, the labor demand elasticity on the lowest paid occupation is finite, and the labor supply elasticity is positive, then, starting from the optimal tax and transfer system with no minimum wage, introducing a binding minimum on occupation 1 (the lowest paid occupation) is desirable if and only if* $g_1 > 1$.

This proposition can also be demonstrated as in Section 3. Starting from the tax optimum with no minimum wage, setting $\bar{w} = w_1$, and increasing c_1 improves social welfare when $g_1 > 1$ without triggering any behavioral response because those who would like to move to occupation 1 cannot do so because of the minimum wage rationing. Those already in occupation 1 are not displaced because of the efficient rationing assumption.

Barring extreme Ralwsian redistributive tastes or no redistributive tastes at all, the government values marginal redistribution toward the lowest paid workers so that $g_1 > 1$. Hence, this proposition constitutes a strong case for introducing a minimum wage even when the government has already implemented the optimal fully general and nonlinear tax and transfer system. It also shows that our results on the desirability of the minimum wage are not due to assuming extensive labor supply responses only.

Theoretically, the occupation model can be seen as a generalized multi-person Ramsey optimal tax model, which inherits most of the structure and properties of the Ramsey model. In particular, the analysis of minimum wages parallels the theory of rationing in second-best Ramsey optimal tax models developed by Guesnerie (1981) and Guesnerie and Roberts (1984). Following Samuelson (1958), the optimal tax system (absent a minimum wage) subsidizes goods going to disadvantaged individuals (here low skilled worker). It is then socially desirable to ration subsidized low skill labor using a minimum wage.

It is important to understand why our positive results contrast with the negative results of Allen (1987) or Guesnerie and Roberts (1987) obtained in the context of the Stiglitz (1982) model of optimal nonlinear taxation. The key difference between the Stiglitz model and the occupation model we use is that, in the Stiglitz model, a minimum wage has no effect on the incentive compatibility constraint because high skilled individuals who imitate low skilled individuals just cut their hours of work but remain in the high skill sector and hence face no unemployment risk due to the minimum wage. Therefore, the minimum wage just hurts low skilled workers through unemployment and without allowing the government to increase their disposable income (as disposable income is constrained by the incentive compatibility constraint). In contrast, in our model, the minimum wage effectively prevents behavioral responses (by rationing low skilled work) and hence has a positive impact on incentive compatibility constraints (either those out of work willing to become low skilled workers or higher paid workers willing to switch to low skilled work). Empirically, many studies have shown that there is a strong participation elasticity among at the bottom of the distribution and the minimum wage would clearly impact the ability of marginal workers to move from in and out the labor force. The Stiglitz model entirely ignores such effects.[29]

- **Optimal Minimum Wage Rationing**

Suppose that the optimum minimum wage $\bar{w}$ covers occupations $i = 1, .., i^*$ (we assumed that occupations were ordered). Then all those occupations pay the same wage $\bar{w}$. As a result, the government can no longer distinguish across those occupations and hence the government is forced to tax (or subsidize) them uniformly so that $c_1 = .. = c_{i^*} = \bar{c}$. We denote by $\bar{T} = \bar{w} - \bar{c}$ the net tax on minimum wage workers.

Again, increasing $\bar{c}$ does not produce any behavioral labor supply response (as occupations $1, .., i^*$ are rationed by the minimum wage). Hence, the government should increase $\bar{c}$ up to the point that $\bar{g} = 1$ where $\bar{g} = (h_1 g_1 + .. + h_{i^*} g_{i^*})/(h_1 + .. + h_{i^*})$ is the average social marginal welfare weight on minimum wage workers. This generalizes Corollary 1 that we obtained in the two-skill model.

Corollary 2 *If a minimum wage is desirable, then the optimal transfer to minimum wage workers is such that the average social marginal welfare weight across minimum wage workers equals one.*

Increasing the minimum wage $\bar{w}$ reduces labor demand for minimum wage occupations

[29] It is debatable whether the response along the intensive margin (higher paid workers cutting down on labor supply and reducing their earnings) goes through the hours of work margin as in the Stiglitz model or through the occupation margin as in our model. It is conceivable that the hours of work margin is predominant in the short-run while the occupation margin is predominant in the long-run. However, the important point is that such intensive margins, either hours or work or occupation, are empirically small relative to participation margins for low skilled workers so that this debate is second order.

$i = 1, .., i^*$. This reduction in minimum wage labor demand will lead to a change in the price of other factors. Suppose the government increases $\bar{w}$ by $d\bar{w}$ while keeping $c = (c_0, .., c_I)$ constant. The direct fiscal effect due to the changes in factor prices $(w_1, .., w_I)$ is zero as $\sum_i h_i \cdot dw_i = 0$ (zero-profit condition).

The involuntary change in occupations due to the minimum wage increase will lead to a fiscal effect and a welfare effect. The fiscal effect is $\sum_i T_i dh_i$ and the welfare effect is $\sum_i \Delta G \cdot dh_i / \lambda$ where ΔG is the difference in social welfare for the marginal workers involuntary displaced from their minimum wage job because of $d\bar{w}$. The optimal minimum wage $\bar{w}$ is set so that those effects cancel out. Unfortunately, it does not seem possible to obtain a closed form formula for this optimum minimum wage in the general case although it is easy to generalize the formulas in the case of purely extensive labor supply responses.

- **Many Consumption Goods and Production Efficiency**

It is also possible to extend the tax model to a situation with many goods. In that context, we can show that the standard theorems of public finance, namely the production efficiency theorem of Diamond and Mirrlees (1971) and the no commodity taxation result of Atkinson and Stiglitz (1976) carry over to the model with optimal minimum wage with taxes and transfers.

The production efficiency theorem implies that, at the joint minimum and tax optimum, there should be production efficiency: producers should maximize profits using pre-tax prices for labor inputs and consumption outputs. This result is trivial to verify in the two skill model and remains true with many labor inputs and many consumption goods. As is well known, the production efficiency result implies that there should be no tariffs in the context of an open economy. Therefore, this important free trade result carries over when the government uses a minimum wage.

The Atkinson and Stiglitz (1976) implies that, if utility functions are separable between consumption goods and labor costs and the sub-utility of consumption is homogenous across all consumers, then the optimum tax/minimum wage system should tax labor only and not impose any differentiated taxes on consumption goods. This result also carries over to the joint tax and minimum wage optimum.

5.3 Numerical Simulations

• Case with no Taxes or Fixed Taxes

We make the following parametric assumptions. (1) We assume a CES production function with elasticity of substitution $\sigma > 0$. (2) We assume constant labor supply elasticities $e_i > 0$ by choosing $P_i(w) = (w/\bar{\theta}_i)^{e_i}$. We assume $(h_1^0, h_2^0) = (1/4, 3/4)$. We assume a CRRA social welfare function $G(u) = (u + B)^{1-\gamma}/(1-\gamma)$ with risk aversion parameter $\gamma > 0$ and where $B > 0$ is a constant that is used to avoid infinitely negative utility or infinite social marginal utility for non-workers.[30] We calibrate the production function so that $(w_1^*, w_2^*) = (1, 3)$ and the labor supply functions so that $(h_0^*, h_1^*, h_2^*) = (0.2, 0.2, 0.6)$ at the no minimum wage equilibrium. We always assume that $e_2 = 0.25$ and $B = 0.5$.

Panel A in Table 1 displays the optimum minimum wage markup over the undistorted market wage w_1^* as well as the involuntary unemployment rate (among all low skilled individuals) under various scenarios for e_1, σ, and γ. The table confirms that the optimum minimum wage is increasing in e_1 (comparing columns (1), (2), (3)), decreasing in σ (comparing columns (4), (5), (6)), and has an inverted U-shape pattern with γ (comparing panels A1, A2, and A3). Optimal minimum wage is small for a high $\gamma = 3$ value.

Panel B in Table 1 illustrates numerically that, starting from a substantial flat rate tax where $\tau_1 = \tau_2 = 0.35$ (and using the same parametrization as in Panel A), the optimal minimum wage is much lower (and is actually useless when $\sigma = e_1 = 0.25$).

• Case with Optimal Taxes

Table 2 provides some numerical simulation illustrations using the same parametrization as in the situation with no taxes and transfers of Table 1. Table 2 shows the optimal tax rates with no minimum wage, and then displays the optimal tax rates, optimal minimum wage markup (and associated unemployment level among the unskilled) in the case of joint minimum wage/tax optimization. The Table confirms our key findings that the minimum wage should be associated with higher low skilled work subsidies than in the case of optimal tax rates with no minimum wage. The table also shows that the optimal minimum wage is increasing with e_1 and decreasing with σ. Finally, the minimum wage is useless in the high redistributive case

[30] B could represent for example a uniform lumpsum transfer, whose cost is not affected by behavioral responses. Next Section will allow for such transfers (along with taxes).

$\gamma = 3$ as $g_1 < 1$ at the pure tax optimum.[31] Interestingly, comparing Tables 1 and 2 suggests that the minimum wage with optimal taxes is not necessarily smaller than in the case with no taxes, especially in the case where redistributive tastes are not too large ($\gamma = 0.5$).

6 Conclusion

Our paper has proposed a theoretical analysis of optimal minimum wage policy for redistribution purposes in a perfectly competitive labor market, considering first the case with no taxes and transfers and then analyzing the case with optimal taxes and transfers. In light of the previous literature on this topic, we find that the standard competitive labor market model offers a surprisingly strong case for using the minimum wage when we make the efficient rationing assumption. A minimum wage is a useful tool if the government values redistribution toward low wage workers and, importantly, this result remains true in the presence of optimal nonlinear taxes and transfers. In that context, our model of occupational choice abstracting from hours of work allows us to overcome the informational inconsistency that has plagued previous work analyzing minimum wage policy with optimal income taxation. Our model fits into the general theory of rationing developed by Guesnerie (1981) and Guesnerie and Roberts (1984) as a minimum wage effectively rations low skilled labor. Such rationing is desirable because the optimal tax/transfer over-encourages the supply of low skilled labor.

We have also derived formulas for the optimal minimum wage in various settings, showing the key economic forces at play. The optimal minimum wage decreases with the demand elasticity for low skilled labor but increases with the supply elasticity of low skilled labor. The optimal minimum wage follows an inverted U-shape as a function of the strength of the redistributive tastes of the government. When labor supply is along the extensive margin, a situation that might approximate well OECD low skilled labor markets, a minimum wage should always be associated with in-work subsidies: the co-existence of minimum wages with high participation tax rates for low skilled workers is (second-best) Pareto inefficient. In that situation, a cut in employer payroll taxes which decreases the gross minimum wage while keeping the net minimum wage constant, combined by an offsetting tax increase on higher

[31] The fact that the minimum wage is zero is in large part the consequence of the two skill model assumption. A model with many skills would generate $g_1 > 1$ at the tax optimum except for extreme Rawlsian redistributive tastes. As discussed below, such a model would allow a better calibration of the percentile of the wage distribution where the minimum wage should hit.

skilled workers is Pareto improving.

There are a number of issues that we have abstracted from in our very stylized model that are worth pointing out as caveats and potential avenues for future research.

First, as mentioned, we abstract from the hours of choice decision which allows us to develop a model with no informational inconsistencies. However, the fact remains that, in practice, taxes and transfers are based on earnings while minimum wages are based on hourly rates. In reality, the government can observe both earnings and hours of work of employees as this information is in general included in the payroll accounting of employers and is actually sometimes required to be reported to the government for administering payroll taxes or maximum hours laws.[32] Therefore, the puzzle remains for why taxes and transfers are based on earnings rather than wage rates. A possible explanation is that hours of work are not very elastic and that most of the labor supply response takes place along the occupation decision and in particular the participation decision. If hours were very elastic, taxes and transfers should be based (at least in part) on wage rates.[33] We conjecture that our results on the desirability of the minimum wage, would carry over to that case as well as long as the supply response along the occupation and participation margin is positive.[34]

Second, a minimum wage rationing mechanism operates very differently from a tax and transfer which alters prices but lets markets clear freely. Our model has assumed that the efficiency costs of each tool should be evaluated using standard welfare theory. It is conceivable, however, that the rationing and hence involuntary unemployment creates additional psychological costs that are not captured in our simple model, and that would make minimum wage policies less attractive in practice. Indeed, democratic states do put limits on quantity policies justified by individual rights that can sometimes conflict with the objective of the government. An extreme example is forced labor through serfdom in the middle ages which was an efficient way (from the perspective of the sovereign) to protect its fiscal base through

[32] For example, in France, hours of work are available in the administrative database DADS used by the social security administration.

[33] Some transfer programs are based partly on hours information. For example, the British Working Families Credit is given only to families where one earner works at least 16 hours a week. Similarly, the current US welfare program TANF imposes work requirements which is an indirect way of conditioning transfers on hours of work.

[34] If this occupational response is zero and the response is fully along the hours margin, then a tax based on wage rates can achieve the first best.

quantity constraints.[35]

Third and related, in order to understand better the real costs of rationing, it could be valuable to consider labor market models with frictions. In particular, search frictions, would allow endogenize job matches and the degree of efficiency in the rationing created by the minimum wages. Therefore, it would be particularly interesting to analyze how our results would carry over in a model with frictions. Hungerbuhler and Lehmann (2007) have made an important step in analyzing optimal minimum wage policy with optimal tax in a search model and it would be interesting to understand more clearly how their results relate to ours.

Fourth, it is conceivable as well that minimum wage policies are favored by the public relative to taxes and transfers because the costs of higher wages at the bottom coming from other factors of productions are not directly visible and hence might be more acceptable to higher income earners than direct taxes to finance transfers toward low income families (this relates to political economy models imposing constraints on redistribution). The government might also perceive as cheaper to increase the minimum wage rather than increasing taxes and transfers. Our results suggest that tax rates on minimum wage workers should not be positive. This could be achieved by exempting from payroll taxes the first bracket of earnings (as is currently done in the United Kingdom for both employee and employer payroll taxes). If the exempt amount is indexed on the minimum wage, then increasing the minimum wage has direct fiscal costs for the government. Interestingly, our results implying that the tax rate on minimum wage workers should be negative implicitly builds such fiscal costs which would make visible the cost of minimum wage increases.

Finally, our numerical simulations have been purely illustrative and it would be worth trying to calibrate the simulations using empirically estimated parameters for the labor demand and supply elasticities, and the degree of efficiency of the rationing created by the minimum wage.

[35] The fiscal explanation for serfdom has been proposed by historians (see e.g., Ardant 1971, 1972) and also fits strikingly well into the theory of Guesnerie and Roberts (1987). A Leviathan government wants to extract as much revenue from its subjects and forbidding people from leaving the state is an effective quantity based policy in that context.

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Figure 1: Desirability of Small Minimum Wage

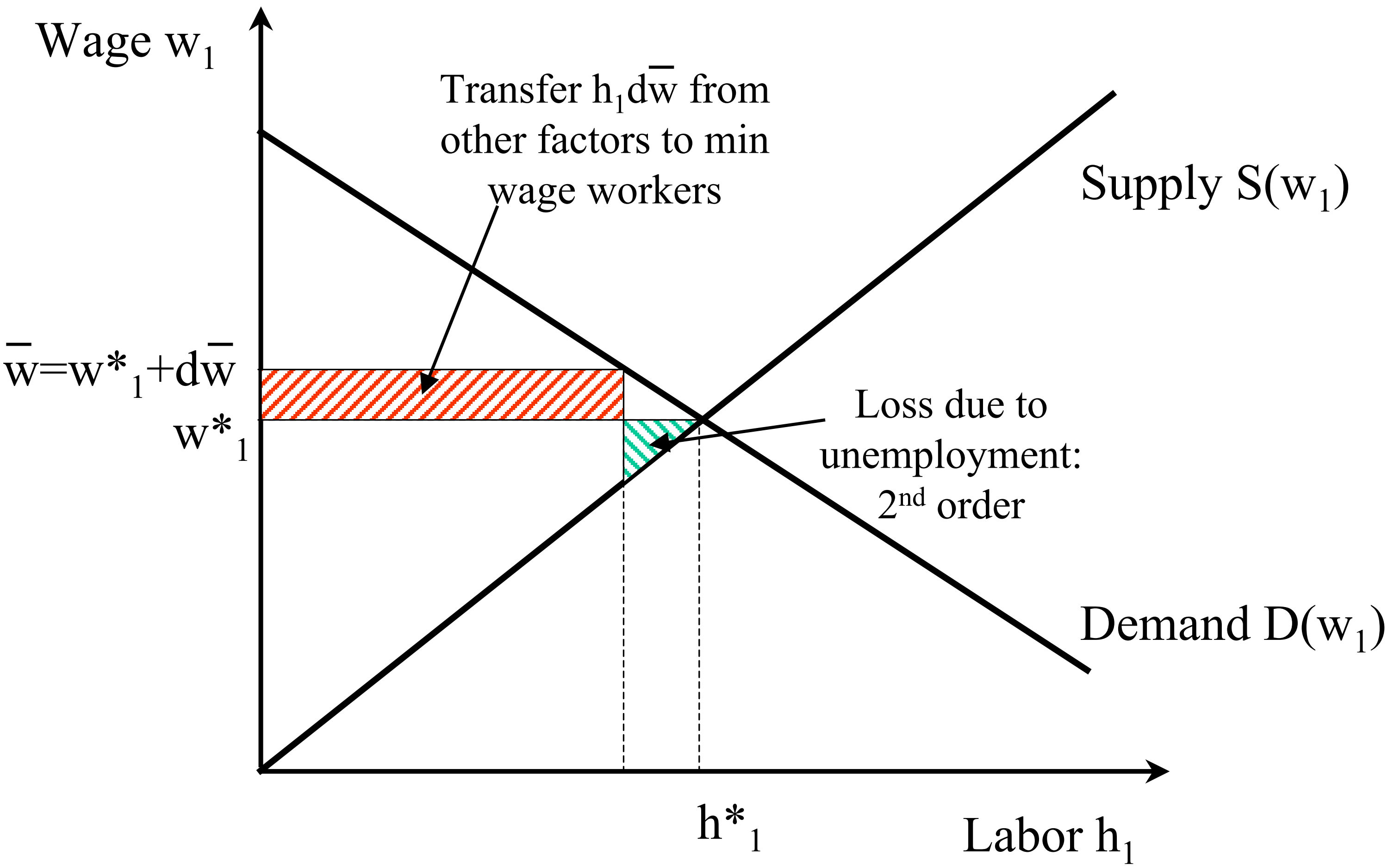

Figure 2: Deriving the Optimal Minimum Wage

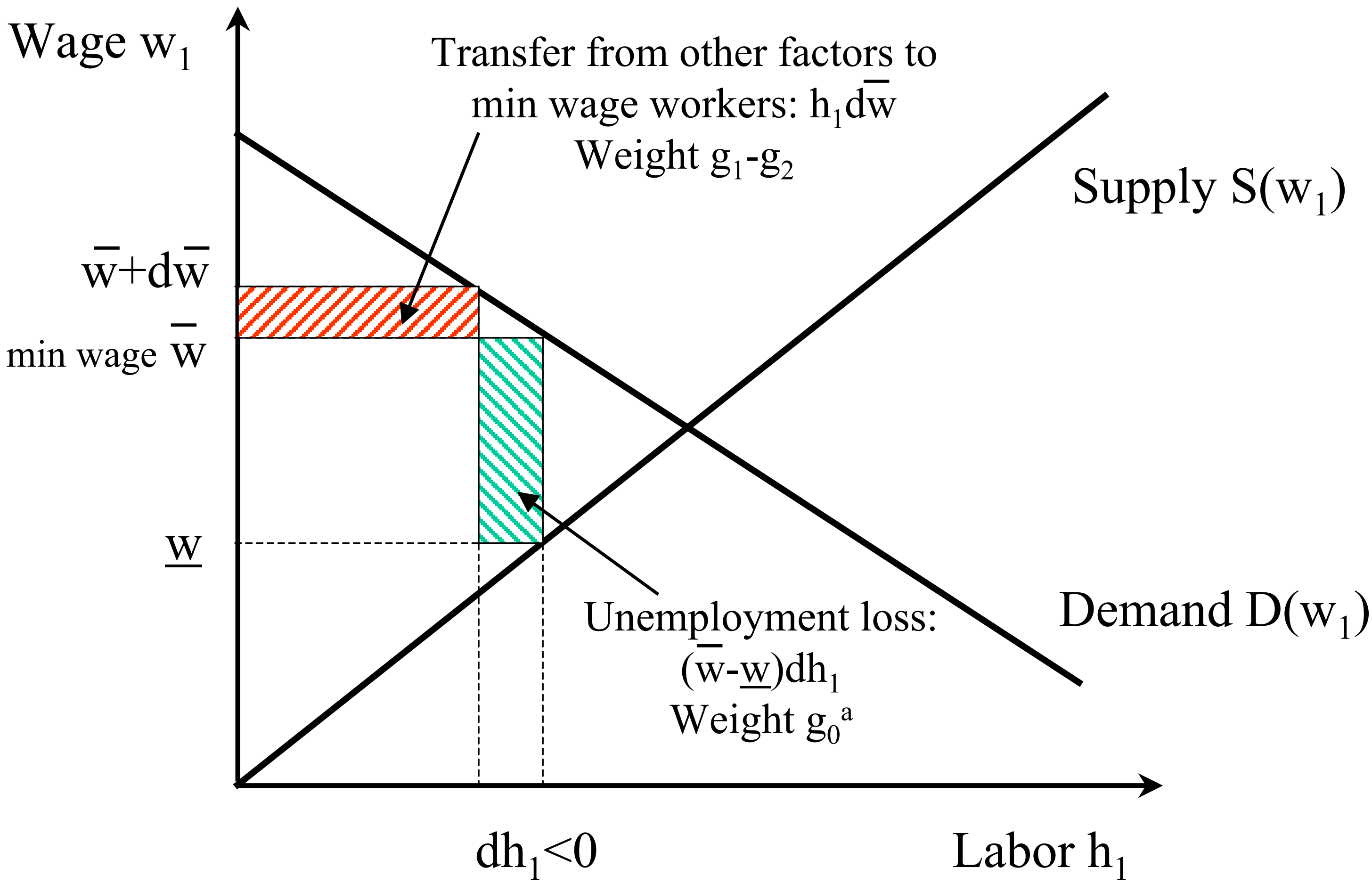

Figure 3: Optimal Minimum Wage with Uniform Rationing

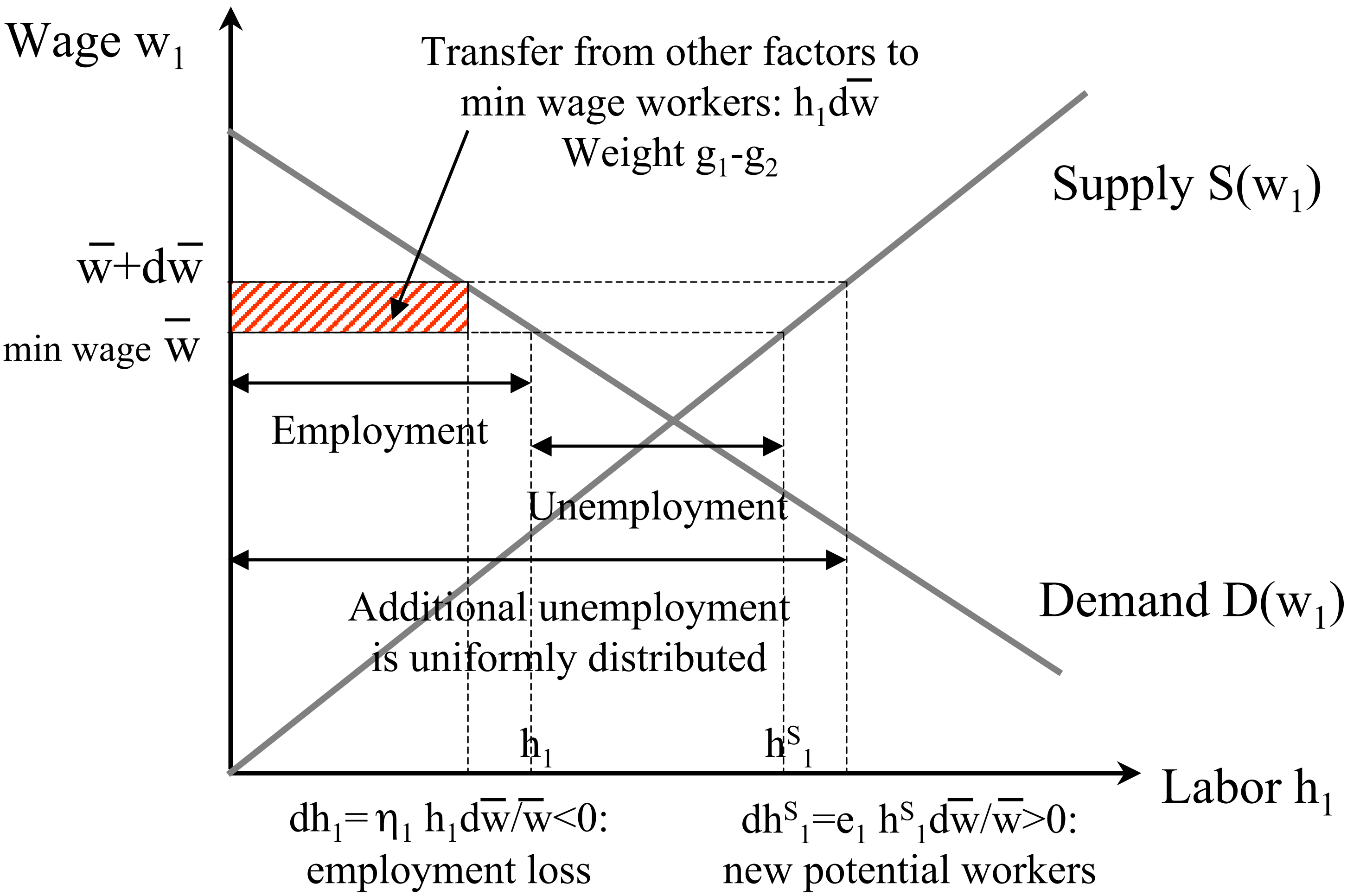

Figure 4a: Optimal Tax/Transfer Derivation

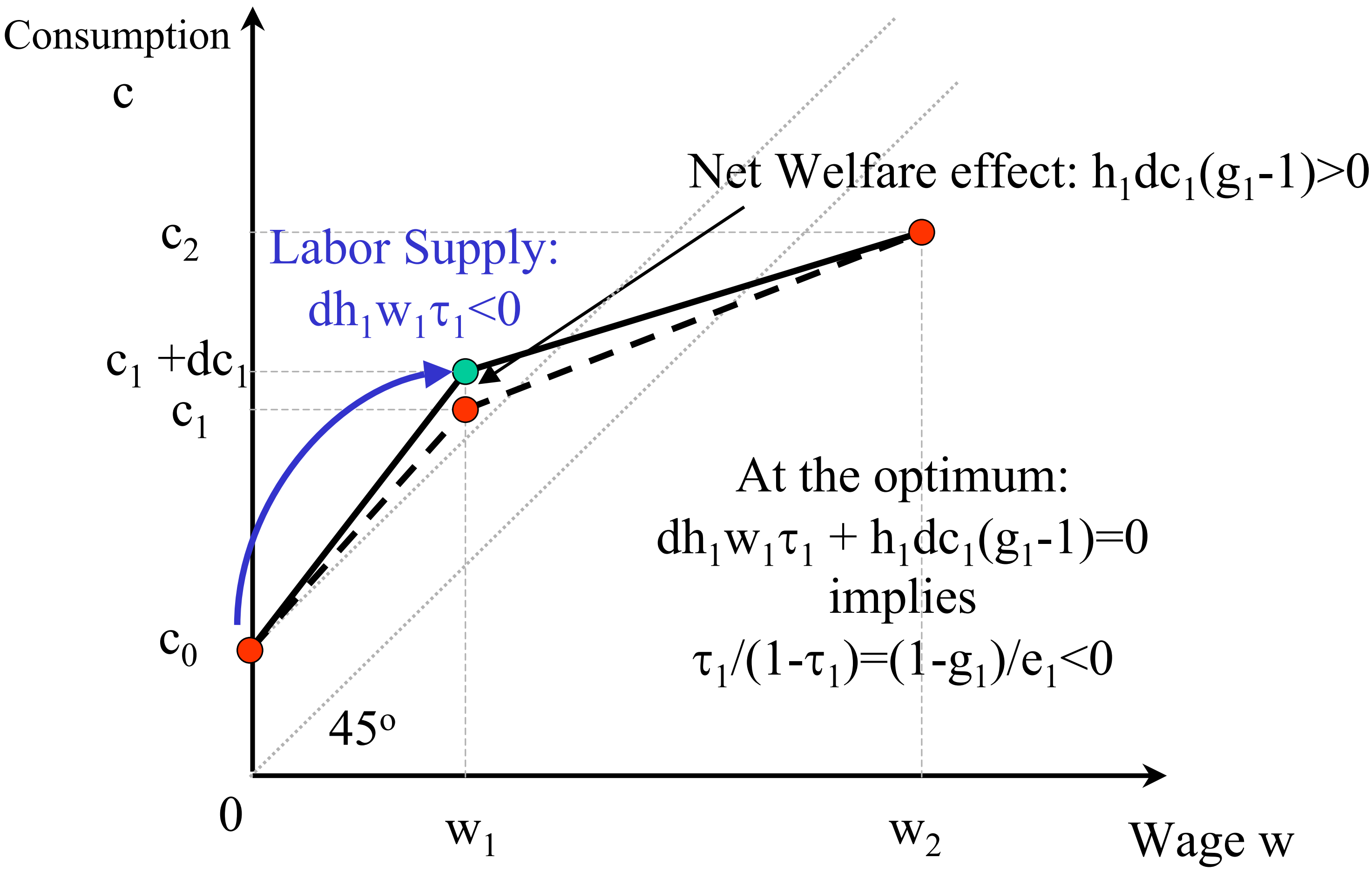

Figure 4b: Optimal Tax/Transfer Derivation

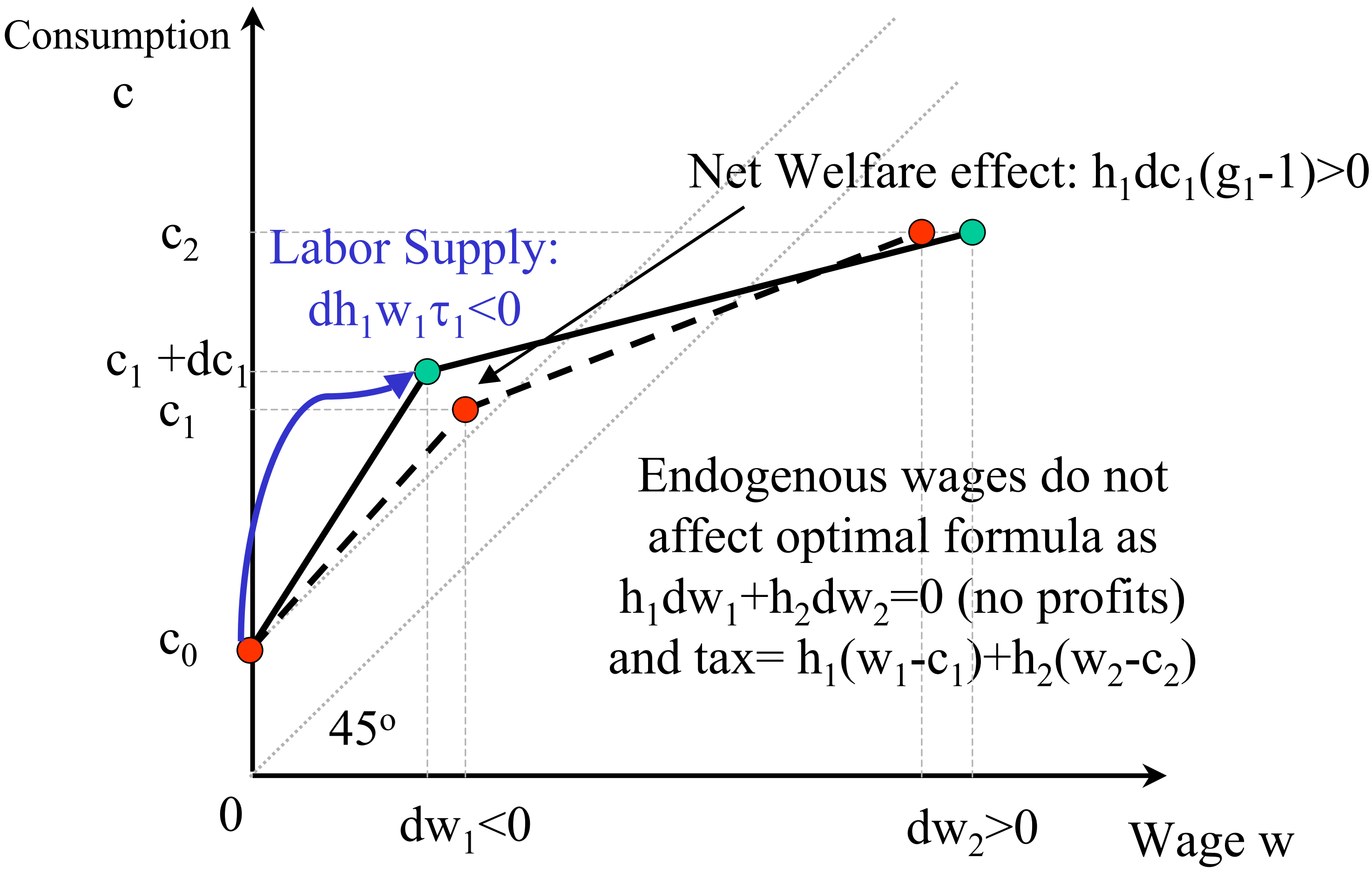

Figure 5: Desirability of Min Wage with Optimal Taxes

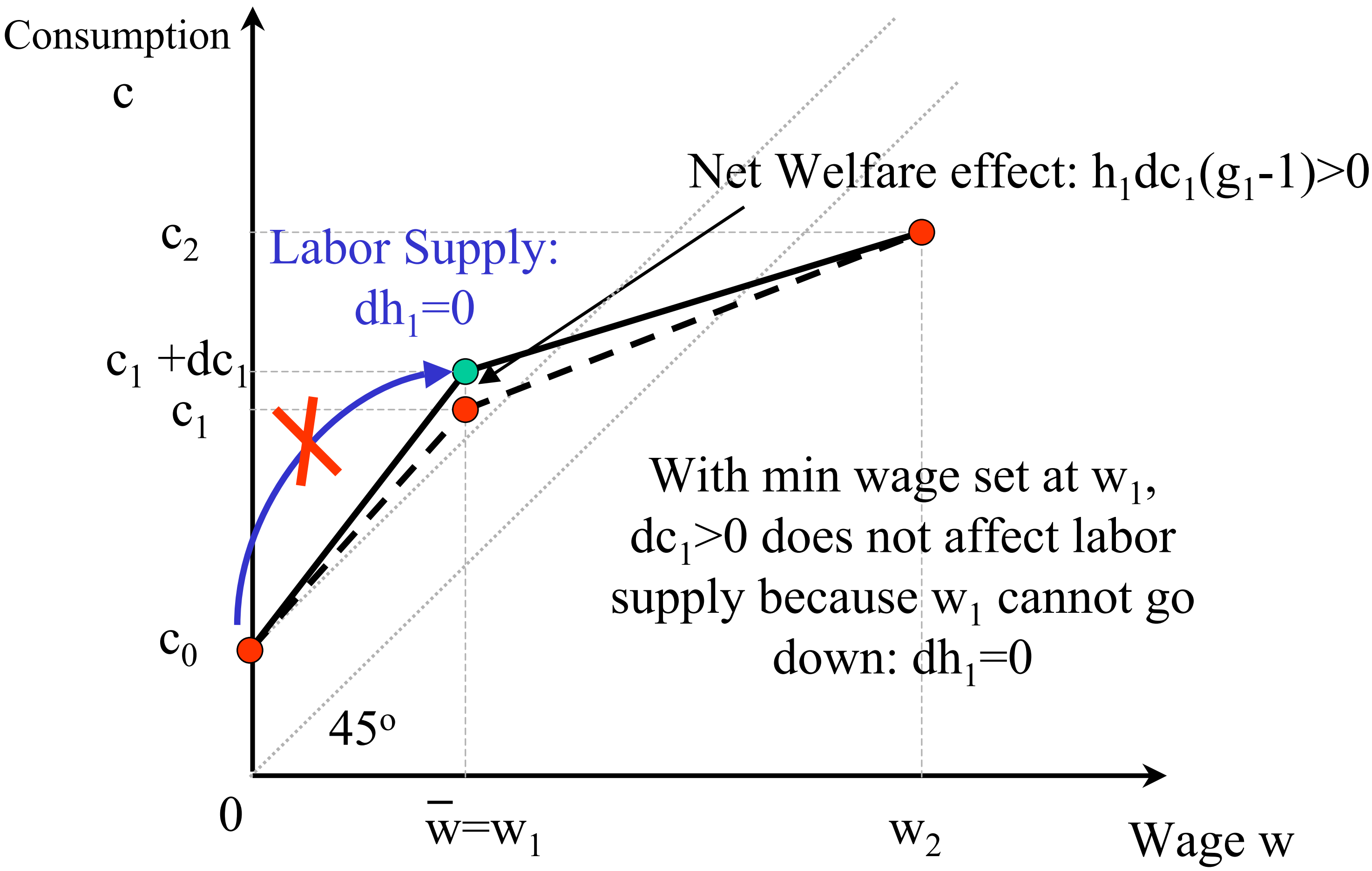

Figure 6: Optimal c_1 with Min Wage such that g_1=1

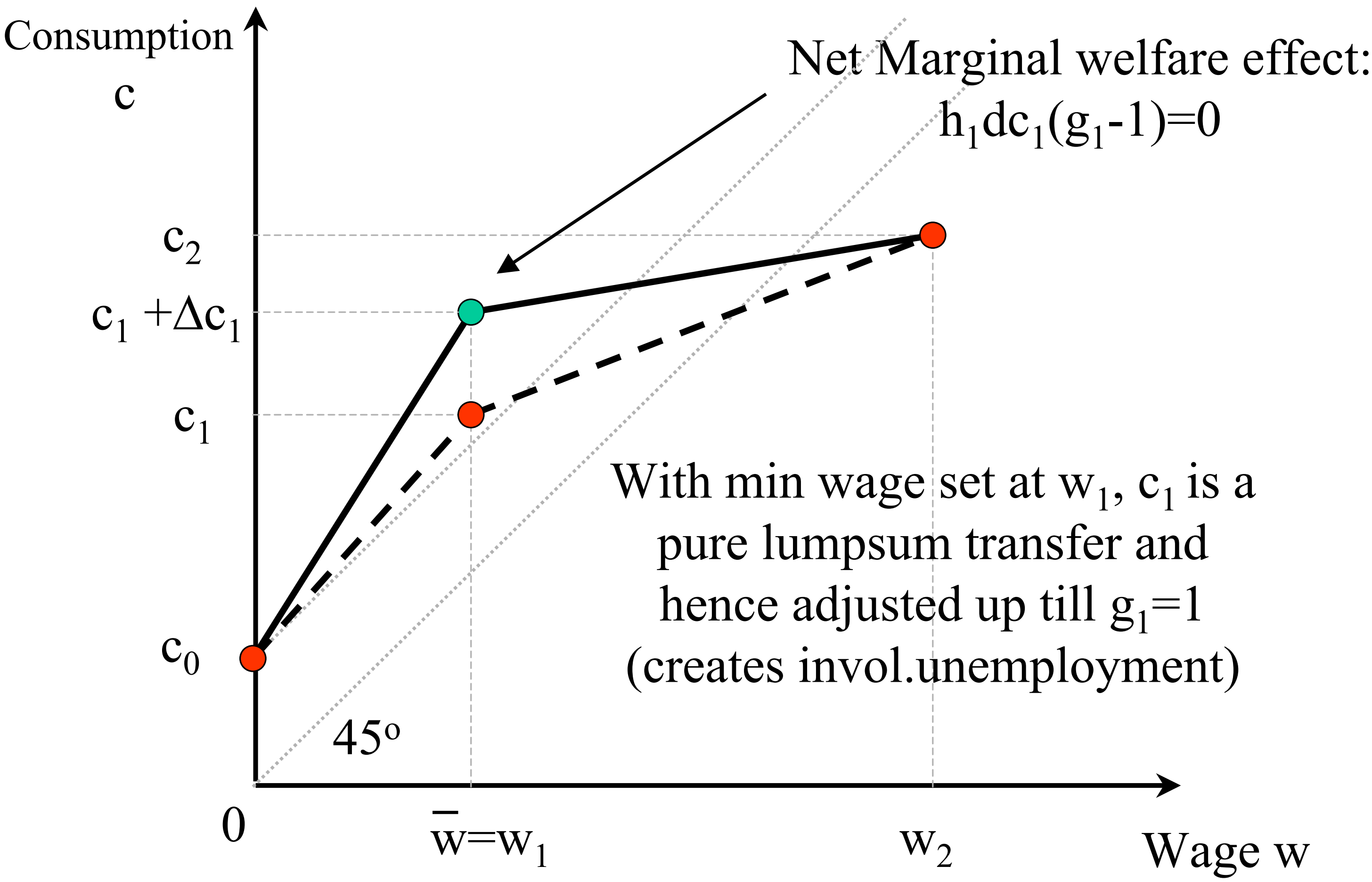

Figure 7: Optimal Minimum Wage Derivation

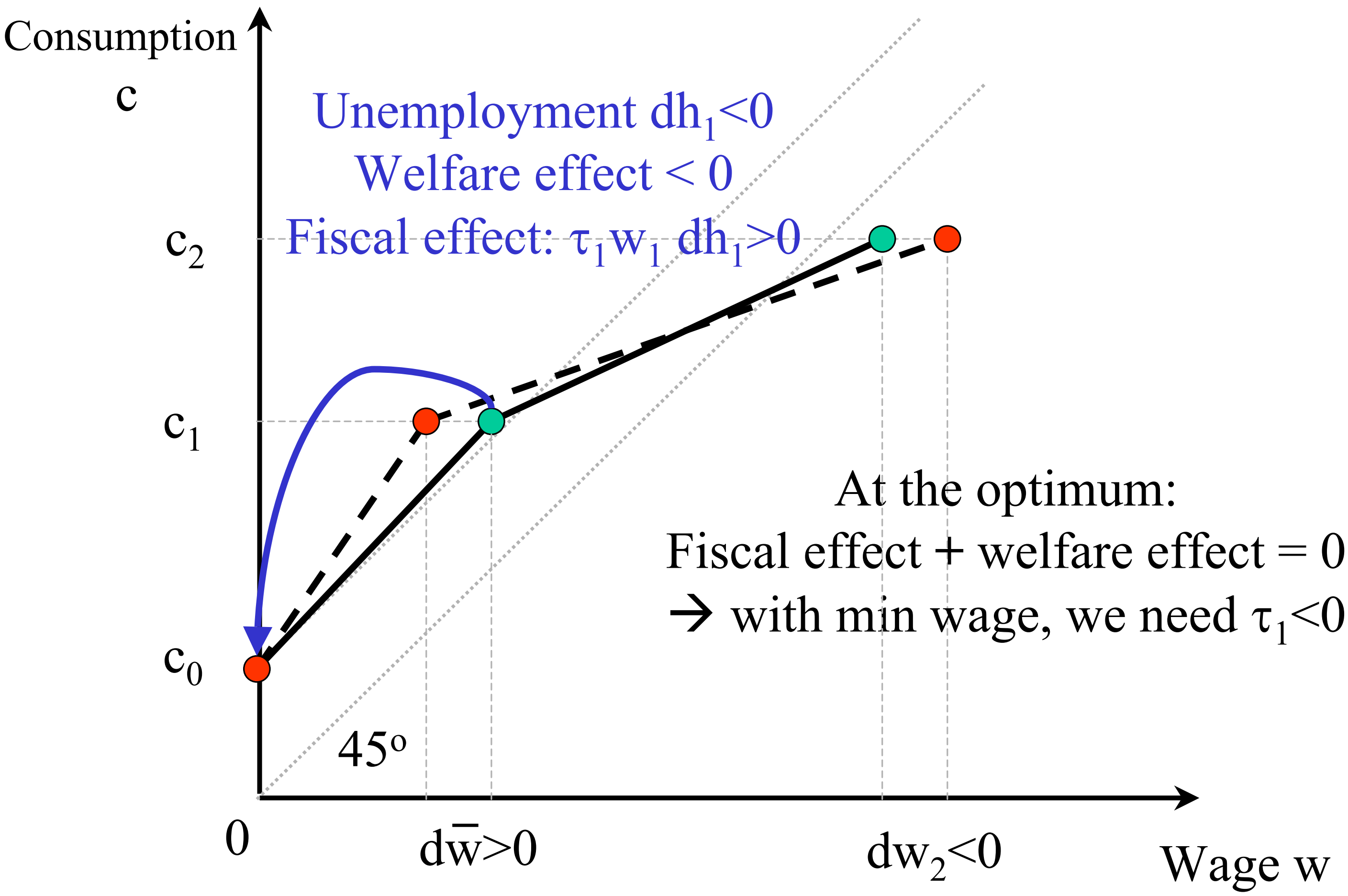

Fig. 8: Double Dividend Policy when τ_1>0 and min wage binds

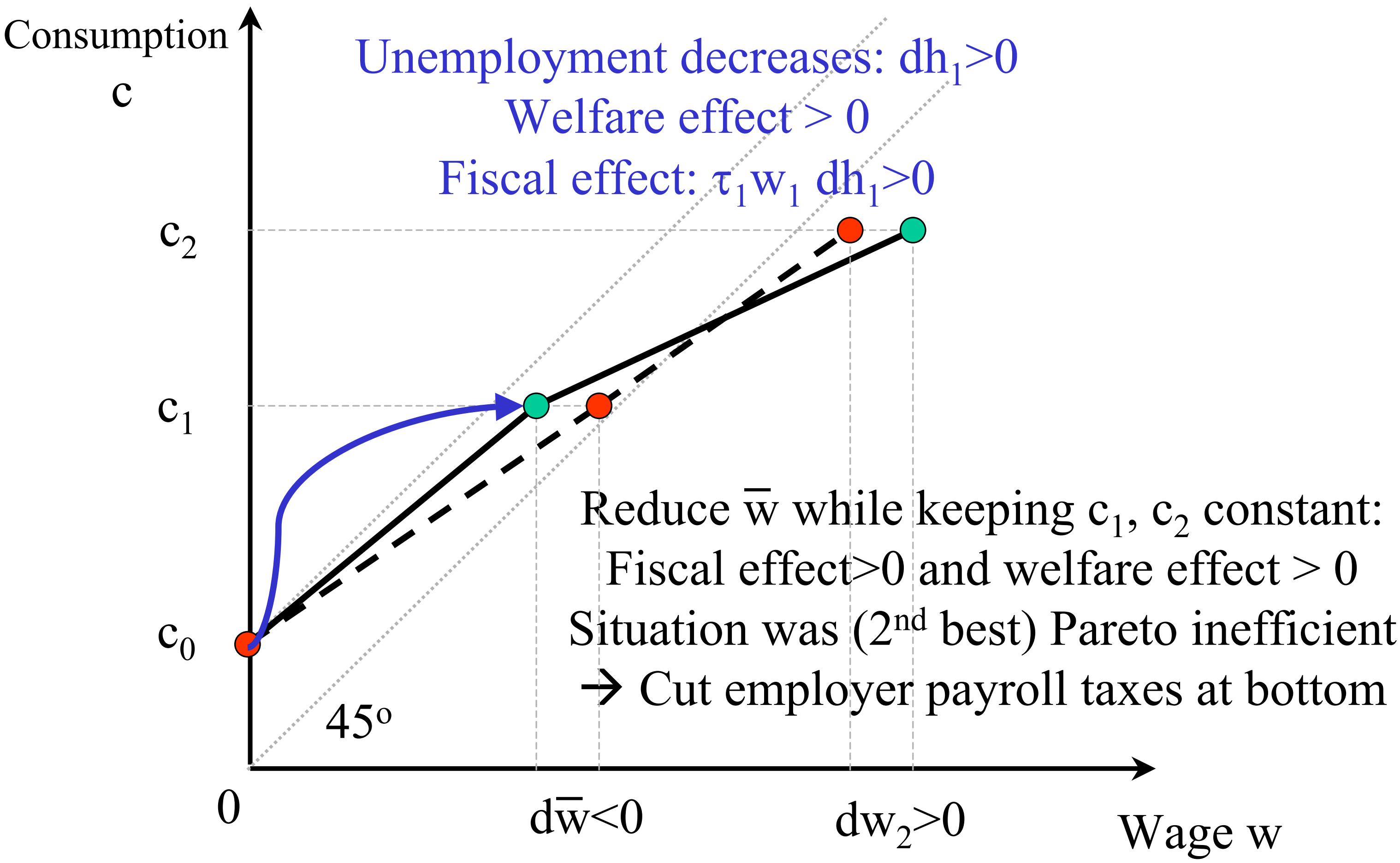

Table 1: Optimal Minimum Wage with No Taxes or Fixed Taxes

	σ=0.5 e_1=0.25	σ=0.5 e_1=0.5	σ=0.5 e_1=1	σ=0.25 e_1=0.5	σ=0.5 e_1=0.5	σ=1 e_1=0.5
	(1)	(2)	(3)	(4)	(5)	(6)
A. Optimum Minimum Wage with no taxes and transfers						
A1. Case γ=1						
Minimum Wage / Market Wage	1.12	1.21	1.34	1.44	1.21	1.07
Unemployment Rate	7.6%	16.8%	39.8%	24.8%	16.8%	9.2%
A2. Case γ=3						
Minimum Wage / Market Wage	1.03	1.08	1.17	1.18	1.08	1.03
Unemployment Rate	2.3%	6.9%	20.1%	10.9%	6.9%	3.8%
A3. Case γ=0.5						
Minimum Wage / Market Wage	1.11	1.19	1.29	1.41	1.19	1.06
Unemployment Rate	6.9%	15.0%	34.9%	23.1%	15.0%	8.1%
B. Optimum Minimum Wage with exogenous taxes (uniform tax rate τ=0.35)						
B1. Case γ=1						
Minimum Wage / Market Wage	1.00	1.01	1.04	1.13	1.01	1.00
Unemployment Rate	0.0%	0.5%	5.5%	8.0%	0.5%	0.0%

Notes: The table reports the minimum wage (relative to market wage rate w^*_1) and the induced unemployment rate among the low skilled as a function of the elasticity of substitution σ between low and high skilled labor in production, the elasticity of labor supply of low skilled workers e_1 (the high skilled labor supply elasticity e_2=0.25 in all cases), and the risk aversion γ of the social welfare function.
The production function is CES with elasticity of substitution γ, calibrated so that market equilibrium with no minimum wage is $(w^*_1,w^*_2)=(1,3)$. The supply functions are calibrated so that $(h^*_0,h^*_1,h^*_2)=(.2,.2,.6)$
The social welfare function is such that $G(u)=(u+0.5)^{1-\gamma}/(1-\gamma)$.

Table 2: Optimal Minimum Wage with Optimal Taxes

	σ=0.5 e_1=0.25	σ=0.5 e_1=0.5	σ=0.5 e_1=1	σ=0.25 e_1=0.5	σ=0.5 e_1=0.5	σ=1 e_1=0.5
	(1)	(2)	(3)	(4)	(5)	(6)
A. Case γ=1						
A1. Optimal Tax Rates with no Minimum Wage						
Tax rate on low skilled workers τ_1	-9.0%	-20.7%	-20.1%	-25.7%	-20.7%	-16.6%
Tax rate on high skilled workers τ_2	45.4%	46.2%	47.6%	47.0%	46.2%	45.4%
A2. Optimal Tax Rates and optimal Minimum Wage						
Tax rate on low skilled workers τ_1	-13.1%	-45.0%	-98.4%	-56.4%	-45.0%	-35.6%
Tax rate on high skilled workers τ_2	45.2%	44.9%	44.2%	45.5%	44.9%	44.3%
Minimum Wage / Market Wage	1.02	1.11	1.31	1.20	1.11	1.06
Unemployment Rate	1.6%	11.2%	52.1%	13.2%	11.2%	9.3%
B. Case γ=3						
B1. Optimal Tax Rates with no Minimum Wage						
Tax rate on low skilled workers τ_1	28.6%	10.2%	-7.6%	5.3%	10.2%	14.1%
Tax rate on high skilled workers τ_2	64.0%	64.2%	64.7%	64.6%	64.2%	63.8%
B2. Optimal Tax Rates and optimal Minimum Wage						
Tax rate on low skilled workers τ_1	28.6%	10.2%	-15.5%	5.3%	10.2%	14.1%
Tax rate on high skilled workers τ_2	64.0%	64.2%	64.4%	64.6%	64.2%	63.8%
Minimum Wage / Market Wage	1.00	1.00	1.05	1.00	1.00	1.00
Unemployment Rate	0.0%	0.0%	6.8%	0.0%	0.0%	0.0%
C. Case γ=0.5						
C1. Optimal Tax Rates with no Minimum Wage						
Tax rate on low skilled workers τ_1	-21.9%	-23.3%	-16.6%	-25.9%	-23.3%	-17.8%
Tax rate on high skilled workers τ_2	32.4%	33.6%	35.2%	34.4%	33.6%	32.0%
C2. Optimal Tax Rates and optimal Minimum Wage						
Tax rate on low skilled workers τ_1	-41.0%	-81.3%	-153.3%	-93.2%	-81.3%	-34.1%
Tax rate on high skilled workers τ_2	31.3%	30.7%	29.1%	31.5%	30.7%	31.1%
Minimum Wage / Market Wage	1.08	1.21	1.49	1.35	1.21	1.04
Unemployment Rate	5.5%	21.9%	91.5%	23.8%	21.9%	4.7%

Notes: The table reports optimal tax rates (on low and high skilled) with no minimum wage and the joint optimal tax rates and minimum wage (relative to market wage rate w^*_1) and the induced unemployment rate among the low skilled as a function of the elasticity of substitution σ between low and high skilled labor in production, the elasticity of labor supply of low skilled workers e_1 (the high skilled labor supply elasticity e_2=0.25 in all cases), and the risk aversion γ of the social welfare function.
The production function is CES with elasticity of substitution γ, calibrated so that market equilibrium with no minimum wage is $(w^*_1,w^*_2)=(1,3)$. The supply functions are calibrated so that $(h^*_0,h^*_1,h^*_2)=(.2,.2,.6)$
The social welfare function is such that $G(u)=(u+0.5)^{1-\gamma}/(1-\gamma)$.